Abstract

Conventional wisdom on public debt management says that liquidity demand should be satiated and that tax rates should be smoothed. Conflicts between the two can arise when government bonds provide liquidity. Smoothing taxes causes greater variability in fiscal balances, and therefore in the supply of government liabilities. When prices are flexible, and can jump to absorb fiscal shocks, the tradeoff between liquidity provision and tax smoothing is eased; when they conflict, optimal policy subordinates tax smoothing to satiating liquidity demand. When price fluctuations impose real costs, conflicts necessarily arise and optimal policy gives primacy to neither goal.

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1 Introduction

Government debt provides liquidity services to the private sector. But, the government must also finance ongoing public expenditures and cope with macroeconomic shocks that have fiscal implications. Optimal debt management must trade off the need for liquidity with these other constraints on fiscal policy. Two well known principles provide the conventional wisdom on debt management: (1) Friedman (1960), abstracting from fiscal considerations, argued that demand for liquidity should be satiated by implementing the celebrated Friedman Rule; seigniorage tax revenues are eliminated. (2) Barro (1979), abstracting from liquidity provision, argued that debt should be used to smooth distortionary tax rates in the short run, albeit at the expense (or benefit) of some long run consumption; fiscal shocks cause consumption and public debt to follow unit root processes.¹

These principles find support in standard models with flexible prices. Chari et al. (1996) recast the Lucas and Stokey (1983) framework with cash and credit goods to a setting with nominal, non-contingent debt. They show that the Friedman Rule is optimal even when a distortionary wage tax is the alternative to seigniorage taxes.² In addition, they show the wage tax rate can be smoothed. Their model includes a third principle of debt management: (3) fiscal shocks should be accommodated by unanticipated jumps in the price level, which represent a non-distortionary tax on existing nominal government liabilities. But in conventional models with monopolistic competition and price rigidities, these unanticipated jumps in the price level, and the ongoing deflation implied by the Friedman Rule, are costly. This leads to a fourth principle of debt management: (4) pursue price stability rather than adhere to the first and third principles.³

¹Aiyagari et al. (2002) formalize the Barro insight in a Ramsey Problem. They discuss when the unit root process emerges in a model with real bonds, no need for transactions balances, and exogenous limits on government assets. They find, among other things, that when the latter are not binding, the Planner will accumulate a war chest of assets so as not to have to use the distortionary tax in the future.
²The principle of uniform taxation favors the use of the wage tax over the seigniorage tax to finance ongoing government expenditures; this principle is discussed further in Sections 3 and 4.
³For example, Woodford (2003) argues that aggregate price level fluctuations create a price dispersion that distorts household consumption decisions. Benigno and Woodford (2003), Schmitt-Grohe and Uribe (2004) and Siu (2004) have shown that the benefits of price stability trump Friedman in such a setting.
This paper asks which of these four principles survive once one recognizes that both money and government bonds provide some degree of liquidity services. Standard analyses assume that the government bonds are “illiquid” in the sense that their supply does not directly affect needed transactions balances, and therefore consumption allocations. For example, in the Friedman tradition, open market operations can be used to satiate the demand for money; liquidity needs are not affected by the decrease in the supply of government bonds. Or in the Barro tradition, a temporary increase in government spending can be bond financed to smooth the path of the tax hikes that will ultimately be needed to service the increase in public debt; again, liquidity needs and consumption allocations are not affected by the path of government bonds. Once one admits that government bonds also provide needed liquidity, optimal debt management may conflict with the Friedman Rule and the smoothing of distortionary taxes.

To investigate these issues, we extend the cash and credit goods model in a natural way to allow for the liquidity of government bonds. In particular, there is a third consumption good – a bond good – that can be purchased by posting government bonds as collateral; the bond good, like the credit good, is actually paid for in the period that follows. Both money and government bonds provide needed transactions balances, and they are in this sense “liquid.”

We will show that if the government has access to a third debt instrument, an illiquid instrument that does not affect needed transactions balances, then the four principles survive largely intact; open market swaps between liquid and illiquid debt can remove conflicts between liquidity management and tax smoothing. In our model with flexible prices, the Ramsey Planner implements an extended Friedman Rule; and the Planner holds the wage tax rate constant over time and across all states of the economy, using unanticipated jumps in the

\cite{Canzoneri2010} provide an extensive review of the conventional wisdom.\footnote{Holmström and Tirole (1998) develop a model in which the private sector does not provide enough liquidity, and the liquidity services of government bonds are welfare improving. Our simple model assumes that only government bonds provide liquidity. Private sector bonds cannot be used as collateral, and are in this sense “illiquid” even though they can be readily bought and sold.}
price level to accommodate fiscal shocks. With price rigidities, the extended Friedman Rule is no longer optimal, but wage tax rates are smoothed in the short run and net government liabilities and consumption exhibit unit root behavior.

When the government does not have access to this third debt instrument, a conflict can arise between satisfying liquidity demand and maintaining a constant tax rate. With flexible prices, the Planner still implements the extended Friedman Rule. When doing so conflicts with holding the wage tax rate constant, the Planner changes the tax rate.\(^5\) We provide examples that illustrate how those conflicts can arise and show how optimal policy reacts when they do. Of more technical interest is the fact that the Ramsey Planner has a meaningful problem in period zero. The Planner will not be tempted to simply inflate away existing nominal liabilities since this would deprive households of needed transactions balances in period zero. With staggered price setting, the short run tax smoothing envisioned by Barro is gone, and the Ramsey solutions are stationary.

Before proceeding to the analysis, we should discuss our assertion that government bonds provide liquidity services. The basic premise should not be controversial. U.S. Treasuries facilitate transactions in a number of ways: they serve as collateral in many financial markets, banks hold them to manage the liquidity of their portfolios, individuals hold them in money market accounts that offer checking services, and importers and exporters use them as transactions balances (since so much trade is invoiced in dollars). The empirical literature finds a liquidity premium on government debt, and moreover the size of that premium depends upon the quantity of debt.\(^6\) Our model captures these facts in a very stylized way, through the artifice of a bond good.

The paper proceeds as follows: Section 2 presents the basic model. Section 3 derives the Ramsey solution for the case in which prices are flexible and the government has access to

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\(^5\)As in Chari et al. (1996) price level jumps make nominal debt state contingent in real terms, but adding a liquidity role for government bonds leads to an important difference in results. Our model do not exhibit the Lucas and Stokey (1983) results with state-contingent debt.

\(^6\)Empirical contributions to this literature include: Friedman and Kuttner (1998), Greenwood and Vayanos (2014), and Krishnamurthy and Vissing-Jørgensen (2012).
the third, illiquid debt instrument; this is a baseline case where the first three principles of
debt management survive largely intact. In Section 4, discusses the Ramsey solution for the
case in which prices are flexible but the government does not have access to an illiquid debt
instrument; here, the extended Friedman Rule is optimal but tax rates may not be constant.
And Section 5 adds monopolistic competition and staggered price setting; here, none of the
first three principles survive. Finally, Section 6 concludes with a summary of our main results
and a discussion of possible interpretations of the third, illiquid debt instrument.

2 A Model with Liquid Government Bonds

The basic structure of our model is easily explained. There are three consumption goods:
a cash good, \( c_m \), a bond good, \( c_b \), and a credit good, \( c_c \). Households face a cash in advance
constraint for the cash good and a collateral constraint for the bond good. Government bonds
are “liquid” in the sense that they can serve as collateral for the bond good. Households pay
for their credit goods and bond goods at the beginning of the period that follows. Firms
produce a perishable final product, \( y \), which can be sold as a cash good, a bond good, or a
credit good. Government purchases, \( g \), are assumed to be credit goods. In each period \( t \),
one of a finite number of events, \( s_t \), occurs. \( s^t \) denotes the history of events, \((s_0, s_1, ..., s_t)\),
up until period \( t \), and \( S^t \) is the set of possible histories. The initial realization, \( s_0 \), is given.
The probability of the occurrence of state \( s^t \in S^t \) is \( \rho (s^t) \).

2.1 Households

The utility of the representative household is

\[
U = \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \rho (s^t) u [c_m (s^t), c_b (s^t), c_c (s^t), n (s^t)]
\]

(1)
where $n_t$ is hours worked. In what follows, we will work with the period utility function

$$
u(\cdots) = \phi_m \log(c_m(s_t)) + \phi_b \log(c_b(s_t)) + \phi_c \log(c_c(s_t)) - \frac{1}{2} n_t^2$$

although our results readily extend to more general preferences.\footnote{In particular our derivation of the Ramsey solution extends to preferences that are homothetic over the three consumption goods and weakly separable across employment and consumption (see Chari et al. (1996)).} This functional form facilitates our exposition: as shown below, it makes the optimal wage tax constant over time and across states in the standard flexible price models (see Scott (2007) and Goodfriend and King (2001)). So, one can interpret movements in that tax rate in our model as exhibiting a tradeoff between tax smoothing and liquidity provision.

Each period is divided into two exchanges. In the financial exchange, public and private agents do all of their transacting except for the actual buying and selling of the final product; purchases of the final good occur in the goods exchange that follows, subject to the cash and collateral constraints:

$$\begin{align*}
M(s_t) &\geq P(s_t) c_m(s_t) \\
B(s_t) &\geq P(s_t) c_b(s_t)
\end{align*}$$

where $P(s_t)$ is the price of the final product, $M(s_t)$ are cash balances, and $B(s_t)$ are government bond holdings; the latter pay a gross rate of return $I(s_t)$ in every future state $s_{t+1}$.

In the financial exchange of period $t$ (or more precisely, state $s_t$), households may purchase state contingent securities, $x(s_{t+1})$, that pay one dollar in state $s_{t+1}$ and cost $Q(s_{t+1}|s_t)$. Households can also issue a nominally riskless bond, $B^c$; this bond is a portfolio of claims that costs one dollar and pays a gross rate of return $I^c(s_t)$ in every future state $s_{t+1}$:

$$1 = I^c(s_t) \sum_{s_{t+1}|s_t} Q(s_{t+1}|s_t)$$

Finally, each household owns a proportionate share of the firms, and receives dividends
Households come into the financial exchange with nominal wealth $A(s^t)$. Their budget constraint is

$$M(s^t) + B(s^t) - B^c(s^t) + \sum_{s^{t+1}|s^t} Q(s^{t+1}|s^t) x(s^{t+1}) \leq A(s^t) \quad (5)$$

and nominal wealth evolves according to

$$A(s^{t+1}) = I(s^t) B(s^t) - I^c(s^t) B^c(s^t) + x(s^{t+1}) + [M(s^t) - P(s^t) c_m(s^t)] - P(s^t) [c_h(s^t) + c_c(s^t)] + [1 - \tau_w(s^t)] W(s^t) n(s^t) + [1 - \tau_Y(s^t)] \Upsilon(s^t) \quad (6)$$

where $\tau_w(s^t)$ and $\tau_Y(s^t)$ are the tax rates on wages and profits.

Households maximize utility (1) subject to the cash and collateral constraints (3), the budget constraint (5) and a no Ponzi game condition.

### 2.2 Firms and Goods Market Clearing

We will begin by assuming that firms are perfectly competitive and prices are flexible. (The penultimate section considers the implications of adding price rigidities.) The representative firm’s production function is

$$y(s^t) = z(s^t) n(s^t) \quad (7)$$

where $z(s^t)$ is a productivity shock common to all firms. $y(s^t)$ can be sold to the households as a cash good, a bond good, or a credit good, or to the government (as a credit good). Firms choose $n(s^t)$ to maximize their after tax profits, $[1 - \tau_Y(s^t)] [P(s^t) y(s^t) - W(s^t) n(s^t)]$ each period.
The goods market clearing condition is

\[ y(s^t) = c_m(s^t) + c_b(s^t) + c_c(s^t) + g(s^t). \] (8)

### 2.3 Government

Government purchases, \( g(s^t) \), are assumed to follow an exogenous process. The government has four policy variables: \( \tau_T(s^t) \) (the tax rate on profits), \( \tau_w(s^t) \) (the tax rate on wages), \( M(s^t) \) (the nominal supply of cash), and \( B(s^t) \) (the nominal supply of government bonds).

The government liabilities \( M(s^t) \) and \( B(s^t) \) are liquid in the sense that they are used by the households to purchase the cash and bond goods. We also consider the case in which the government has access to a third debt instrument that is not liquid in this sense. That is, the government may be able to purchase (and subsequently sell) \( B^c(s^t) \), the nominally riskless asset issued by households; the government’s holdings of this asset will be denoted by \( B^c_g(s^t) \).

### 2.4 Competitive Equilibrium

A competitive equilibrium is an initial nominal asset position \( A_0 \) and sequences of prices

\[
\left\{ P(s^t), W(s^t), Q(s^{t+1}|s^t), I^c(s^t) \right\}_{s^t \in S^t, s^{t+1} \in S^{t+1}} \bigg|_{t=0}^{\infty},
\]

allocations

\[
\left\{ n(s^t), c_m(s^t), c_b(s^t), c_c(s^t), y(s^t), B^c(s^t), B(s^t), M(s^t), A(s^{t+1}) \right\}_{s^t \in S^t, s^{t+1} \in S^{t+1}} \bigg|_{t=0}^{\infty},
\]

and policy variables

\[
\left\{ \{\tau_w(s^t), \tau_T(s^t), g(s^t), I(s^t), B^c(s^t)\} \right\}_{s^t \in S^t} \bigg|_{t=0}^{\infty}
\]

such that:

i) given \( A_0 \), the prices \( \left\{ P(s^t), W(s^t), Q(s^{t+1}|s^t), I^c(s^t) \right\}_{s^t \in S^t, s^{t+1} \in S^{t+1}} \bigg|_{t=0}^{\infty}, \) and the policy variables \( \{\tau_w(s^t), I(s^t)\}_{s^t \in S^t} \bigg|_{t=0}^{\infty}, \) the sequences

\[
\left\{ n(s^t), c_m(s^t), c_b(s^t), c_c(s^t), B^c(s^t), B(s^t), M(s^t), A(s^{t+1}) \right\}_{s^t \in S^t, s^{t+1} \in S^{t+1}} \bigg|_{t=0}^{\infty}
\]
solve the household’s optimization problem,

ii) given the prices \( \left\{ P(s^t), W(s^t) \right\}_{s^t \in S^t} \bigg|_{t=0}^{\infty}, \) and policy variables \( \{\tau_T(s^t)\}_{s^t \in S^t} \bigg|_{t=0}^{\infty}, \) the sequences \( \left\{ n(s^t), y(s^t) \right\}_{s^t \in S^t} \bigg|_{t=0}^{\infty} \) solve the firm’s optimization problem, and
2.4.1 The household’s first order conditions

The representative household maximizes (1) subject to (3) and (5). One first order condition gives the pricing equation for state contingent claims

\[ Q(s^{t+1}|s^t) = \beta \rho(s^{t+1}|s^t) \left[ \frac{u_m(s^{t+1})}{u_m(s^t)} \frac{P(s^t)}{P(s^{t+1})} \right] \]  

(9)

The remaining first order conditions are

\[ u_m(s^t) = I^c(s^t) u_c(s^t) \]  

(10)

\[ u_b(s^t) = (1 + I^c(s^t) - I(s^t)) u_c(s^t) \]  

(11)

\[ -u_n(s^t) = (1 - \tau_w(s^t)) \left( \frac{W(s^t)}{P(s^t)} \right) u_c(s^t), \]  

(12)

the complimentary slackness conditions associated with the cash and collateral constraints (3), and a transversality condition.

These equations are easily interpreted: If the household gives up one dollar’s worth of the cash good, it can spend \( I^c(s^t) \) dollars on the credit good, because credit goods avoid the cash in advance constraint; similarly, if the household gives up one dollar’s worth of the bond good, it can spend \( 1 + I^c(s^t) - I(s^t) \) dollars on the credit good, because credit goods avoid the collateral constraint.

\( I^c(s^t) - 1 > 0 \) is the traditional seigniorage tax on cash, and the traditional Friedman Rule is to set \( I^c(s^t) = 1 \). \( I^c(s^t) - I(s^t) > 0 \) is a second seigniorage tax on liquid bonds. The

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8Using (4), one arrives at the standard Euler equation

\[ 1 = I^c(s^t) \beta E_t \left[ \frac{u_m(s^t)}{u_m(s^t)} \frac{P(s^t)}{P(s^{t+1})} \right] \]
“extended Friedman Rule” alluded to above is to set $I_c(s^t) = I(s^t) = 1$, eliminating both seigniorage taxes, and satiating the demand for both cash and liquid bonds. The extended Friedman Rule carries Friedman’s original intent over to our new economic environment; in what follows, we will simply refer to it as “the Friedman Rule.”

The third equation is the standard labor-leisure tradeoff, which appears to be undistorted by the seigniorage taxes. However, it should be noted that these three equations imply

$$-u_n(s^t) = \left(1 - \tau_w(s^t)\right) \left(\frac{W(s^t)}{P(s^t)}\right) u_m(s^t)$$

$$= \left(\frac{1 - \tau_w(s^t)}{1 + (I_c(s^t) - I(s^t))}\right) \left(\frac{W(s^t)}{P(s^t)}\right) u_b(s^t)$$

(13)

So, the seigniorage taxes do distort the labor-leisure margins. And only the wage tax, $\tau_w(s^t)$, taxes these margins uniformly. This fact will play an important role in what follows.

### 2.4.2 The firm’s first order conditions (with flexible prices)

Firms maximize after tax profits, taking the real wage rate as given. The representative firm’s first order condition is $\frac{W(s^t)}{P(s^t)} = z(s^t)$.

### 3 Flexible Prices with an Illiquid Debt Instrument

We begin with a case in which firms are perfectly competitive, prices are flexible and the government has access to an illiquid debt instrument, $B^c_g(s^t)$; that is, the government can buy or sell the nominally riskless bond issued by households, $B^c(s^t)$.

This is a baseline case in which the first three principles of debt management survive essentially intact. The Friedman Rule is optimal, the wage tax rate is constant, and unanticipated jumps in the price level absorb fiscal shocks. We begin by stating the Ramsey Planner’s problem. We then present the Planner’s optimality conditions, and discuss each of the first three principles of debt management in turn.
3.1 The Ramsey Problem

The implementability conditions and the feasibility constraint are set out in the following lemma.

**Lemma 1:** The flow implementability conditions are

\[ \omega (s^t) = \beta \sum_{s^{t+1} \mid s^t} \rho [s^{t+1} \mid s^t] \omega (s^{t+1}) + 1 - [n (s^t)]^2 \]  

(14)

where

\[ \omega (s^t) \equiv \left[ \frac{\phi_m}{c_m (s^t)} \right] \left[ \frac{A (s^t)}{P (s^t)} \right] = \left[ \frac{\phi_m}{c_m (s^t)} \right] [m (s^t) + b (s^t) - b^c (s^t)] \]  

(15)

and

\[ \frac{c_c (s^t)}{\phi_c} \geq \frac{c_b (s^t)}{\phi_b} \geq \frac{c_m (s^t)}{\phi_m} \].  

(16)

The feasibility constraint is

\[ c_m (s^t) + c_b (s^t) + c_c (s^t) + g (s^t) = z (s^t) n (s^t) . \]  

(17)

The proof is in the online Appendix. Note that small letters are now denoting the real values of assets; that is, \( m (s^t) = M (s^t) / P (s^t) \); \( b (s^t) = B (s^t) / P (s^t) \); \( b^g (s^t) = B^g (s^t) / P (s^t) \).

Equation (14) is a sequence of implementability conditions that is obtained by using the household’s first order conditions to eliminate relative prices and tax rates in the household’s flow budget constraints. It is convenient to write these conditions in terms of \( \omega (s^t) \), which is the value of the government’s net liabilities expressed in terms of utility.\(^9\)

\(^9\)Note that \( \frac{\phi_m}{c_m (s^t)} \) is the marginal utility of the cash good.
implementability conditions (16) are implied by the constraints $1 \leq I(s^t) \leq I^c(s^t)$ and the household’s first order conditions.

**The Ramsey Problem:** The Ramsey Planner chooses a sequence of allocations

$\{c_m(s^t), \ c_b(s^t), \ c_c(s^t), \ n(s^t), \ m(s^t), \ b(s^t), \ b'_g(s^t)\}_{t=0}^{\infty}$ that maximizes household utility, (2), subject to (14) (with Lagrange multipliers $\lambda(s^t)$), (16) (with multipliers $\eta^c(s^t)$ and $\eta^m(s^t)$ respectively), (17) (with multipliers $\mu(s^t)$) and the cash and collateral constraints (with multipliers $\gamma^m(s^t)$ and $\gamma^b(s^t)$ respectively).

### 3.2 The Planner’s Optimality Conditions and Period Zero

The Planner’s first order conditions for periods $t \geq 0$ are:

\[ c_c(s^t) : \quad \frac{\phi_c}{c_c(s^t)} = \mu(s^t) - \frac{\eta^c(s^t)}{\phi_c} \]  \hspace{1cm} (18)

\[ c_b(s^t) : \quad \frac{\phi_b}{c_b(s^t)} = \mu(s^t) + \gamma^b(s^t) + \frac{\eta^c(s^t) - \eta^m(s^t)}{\phi_b} \]  \hspace{1cm} (19)

\[ c_m(s^t) : \quad \frac{\phi_m}{c_m(s^t)} = \mu(s^t) + \gamma^m(s^t) \left[ 1 - \frac{\omega(s^t)}{\phi_m} \right] + \frac{\eta^m(s^t)}{\phi_m} \]  \hspace{1cm} (20)

\[ n(s^t) : \quad n(s^t) = \frac{\mu(s^t) z(s^t)}{1 + 2\lambda(s^t)} \]  \hspace{1cm} (21)

\[ b(s^t) : \quad \gamma^m(s^t) = \gamma^b(s^t) \equiv \gamma(s^t) \]  \hspace{1cm} (22)

\[ b'_g(s^t) : \quad \gamma(s^t) = 0 \]  \hspace{1cm} (23)

\[ \omega(s^{t+1}) : \quad \lambda(s^{t+1}) - \lambda(s^t) = \gamma(s^{t+1}) \frac{c_m(s^{t+1})}{\phi_m} \]  \hspace{1cm} (24)

where substitutions have eliminated $m(s^t)$ as a choice variable, and the transversality condition

\[ \lim_{n \to \infty} \beta^n \sum_{s^{t+n}|s^t} \rho(s^{t+n}|s^t) \omega(s^{t+n}) = 0. \]  \hspace{1cm} (25)
The multiplier $\mu(s^t)$ is the shadow price of a unit of output, while $\eta^c(s^t)$ is the shadow price of the first inequality in (16); increasing $c_c(s^t)$ eases this constraint. So, (18) says to increase consumption of the credit good beyond the cost of an additional unit of output. The $\eta$ terms in (19) and (20) play similar roles. $\gamma^m(s^t)$ and $\gamma^b(s^t)$ are the shadow prices of the cash and collateral constraints, and cutting back on cash and credit good consumption eases these constraints; (19) and (20) say to do just that.

The last three conditions describe optimal money and debt management. The choice of $\omega(s^t) \equiv (m(s^t) + b(s^t) - b^c_g(s^t)) (\phi_m/c_m)$ sets the level of net government liabilities. The Lagrange multiplier on the implementability constraint, $\lambda(s^t)$, represents the shadow price of the debt burden. If the liquidity constraints are binding ($\gamma(s^{t+1}) > 0$), then (24) implies that $\lambda(s^{t+1}) - \lambda(s^t) > 0$.

A familiar problem arises in period zero. The natural state variable in our setup is $A(s^t) = M(s^t) + B(s^t) - B^c_g(s^t)$, the net nominal liabilities of the government; this suggests that the initial condition should be placed on $A(s^0)$. But then, the Planner would be tempted simply to inflate away its old debt to the private sector, and inject new transactions balances in exchange for $B^c_g(s^0)$. There is really no meaningful period-zero optimization when the government has access to the illiquid debt instrument, and thus there is no theory of the optimal level of government liabilities for this version of the model.\textsuperscript{10} This problem is well understood in the literature. To set aside these issues, we will modify the Ramsey problem at date zero. Specifically, the Ramsey planner sets the real values $m(s^0), b(s^0), \text{ and } b^c_g(s^0)$, subject to a given (possibly zero) value for real net government liabilities, $m(s^0) + b(s^0) - b^c_g(s^0)$.

The following proposition summarizes optimal policy.

**Proposition 1:** When prices are flexible and the government can actively trade an illiquid bond, optimal policy is characterized by (1) the Friedman Rule, (2) constant tax rates, and (3) unanticipated price level jumps that equate the real value of nominal liabilities.

\textsuperscript{10}This is an example of the familiar problem of multiple steady states under optimal policy. Initial conditions will determine real liabilities.
to the present value of primary surpluses, thereby absorbing fiscal shocks.

We discuss each of these in turn. Proofs are in the online Appendix.

3.3 Primacy of the Friedman Rule

The first order condition for $b^c_g (s^t)$, (23), sets $\gamma (s^t) = 0$ in any state and at any time. The liquidity constraints are never binding. The intuition is that the Planner can always provide the necessary gross liabilities by exchanging $b^c_g (s^t)$ for $m (s^t) + b (s^t)$, and then provide the required liquidity for the cash and collateral constraints individually by using the usual open market operations. As shown in the online Appendix, $\eta^c (s^t)$ and $\eta^m (s^t)$ are always equal to zero in this case; the inequality constraints (16) are not binding. Since $\gamma (s^t)$ is also equal to zero, the first order conditions (18), (19) and (20) imply that

$$\frac{\phi_m}{c_m (s^t)} = \frac{\phi_b}{c_b (s^t)} = \frac{\phi_c}{c_c (s^t)}$$

(26)

And from the households’ first order conditions, (10) and (11), $I^c (s^t) = I (s^t) = 1$. The Friedman Rule sets the marginal rates of substitution to one (the marginal rate of transformation), as is required by efficiency, and eliminates both seigniorage taxes. The tax on labor income raises any revenue that is needed. The fact that the Planner uses only the wage tax is a reflection of the superiority of uniform taxation when preferences are homothetic in the consumption goods and separable in consumption and leisure (Atkinson and Stiglitz (1976) and Chari et al. (1996)). (12) and (13) show the labor-leisure margins; the tax on labor income distorts all three uniformly, while the seigniorage taxes do not.

The following lemma will be useful in understanding the implications of the optimality of the Friedman Rule.

**Lemma 2:** When the Friedman Rule holds, the wage tax rate becomes

$$1 - \tau_w (s^t) = \frac{1}{1 + 2\lambda (s^t)}$$

(27)
The proof of Lemma 2 is in the online Appendix.

3.4 The Optimality of Tax Smoothing

Since $\gamma(s^t) = 0$, the first order condition for $\omega(s^t)$ then implies that $\lambda(s^t) = \lambda$, a constant in all states and at any time. From (27) this implies that $\tau_w(s^t)$ is constant and smooths the distortion it creates. The wage tax rate is set at a level that will finance the initial government liabilities and average government expenditures.

3.5 The Price Level

If there are no revenues from the seigniorage taxes and the wage tax rate is constant, how does the Ramsey Planner deal with fiscal shocks? The Planner can engineer a jump in the price level that changes the real value of existing nominal liabilities: a surprise inflation (or deflation) is a non-distortionary tax (or subsidy) on nominal balances, and it has no cost in our flexible price models. Suppose for example that there is an unexpected increase in government spending. This will decrease the present discounted value of fiscal surpluses. The Planner would simply increase the price level, lowering the real value of existing nominal liabilities, and keeping the present value budget constraint in balance.

3.6 Summing Up

The bottom line is that the Planner always implements the Friedman Rule, and doing so does not conflict with a constant wage tax rate. Simply put, our baseline case is a natural extension of the standard model, and the first three principles of conventional debt management survive the introduction of a liquidity role for government bonds. Public finance – that is, the running of surpluses and deficits – determines the level of net liabilities, $m(s^t) + b(s^t) - b^g_g(s^t)$. But by buying and selling the illiquid debt instrument, the Planner can control the level of gross liabilities, $m(s^t) + b(s^t)$, that are needed for liquidity; then,
ordinary open market operations can set the right mix of cash and liquid bonds. The Planner thereby avoids any conflict between liquidity provision and tax smoothing.

4 Flexible Prices, But No Illiquid Debt Instrument

Here we consider the more interesting case in which the government does not have access to the illiquid private sector bond; that is, \( b^g (s^t) \equiv 0 \) and \( \omega (s^t) \equiv (m (s^t) + b (s^t)) (\phi_m / c_m (s^t)) \).

In this case, the felicitous outcome described in Section 3 may fall apart, and the first three principles of conventional debt management may not all survive. The most obvious observation is that, without the first order condition for \( b^g (s^t) \), one can no longer conclude at the outset that the liquidity constraints will never be binding.

4.1 The Ramsey Problem

The Ramsey problem is nearly identical to that in Section 3, but with one fewer policy instrument. The Ramsey Planner’s problem is to choose a sequence of allocations \( \{ c_m (s^t), c_b (s^t), c_c (s^t), n (s^t), m (s^t), b (s^t) \}_{t=0}^{\infty} \) that maximizes household utility, \( (2) \), given an \( A (s^0) > 0 \) and subject to (14) (with Lagrange multipliers \( \lambda (s^t) \)), (16) (with multipliers \( \eta^c (s^t) \) and \( \eta^m (s^t) \) respectively), (17) (with multipliers \( \mu (s^t) \)) and the cash and collateral constraints themselves (with multipliers \( \gamma^m (s^t) \) and \( \gamma^b (s^t) \) respectively).

The optimality conditions are (18) - (22), (24), and (25). And (27) still expresses the optimal wage tax rate in terms of the shadow price of debt.

The following lemma, which is proved in the online Appendix, will be useful in what follows:

**Lemma 3:** If \( I^c (s^t) = I (s^t) = 1 \), the cash in advance and bond collateral constraints can be combined into a consolidated Planner’s liquidity constraint:

\[
\omega (s^t) \geq \phi_m + \phi_b.
\]
And the constraint holds with equality when $\gamma(s^t) > 0$.\footnote{Below, it is shown that the Friedman Rule is indeed optimal.}

In our model, there is a meaningful Ramsey problem in period 0. As noted above, the natural state variable in our model is $A(s^t) = M(s^t) + B(s^t)$, the nominal value of government liabilities. Here, the Ramsey Planner inherits some $A(s^0) > 0$. The Planner’s choice of $P(s^0)$ determines the real value of government liabilities in period zero and open market operations determine the optimal mix of liquidity between money, $m(s^0)$, and bonds, $b(s^0)$. The Planner will not be tempted to inflate away initial liabilities because households would not have the liquidity they need in period zero. So here we truly have a theory of the optimal level of public debt.

The following lemma is proved in the online Appendix:

**Lemma 4:** The Ramsey solution in period zero sets

$$
\gamma(s^0) \frac{c_m(s^0)}{\phi_m} = \lambda(s^0). \tag{29}
$$

The Friedman Rule is optimal in period zero, $I^c(s^0) = I(s^0) = 1$.

In period 0, (29) equates the shadow value of initial liquidity (measured in terms of the cash good), $\gamma(s^0)$, to the shadow cost of the future tax liabilities, $\lambda(s^0)$. And since $\lambda(s^0) > 0$, it must be that $\gamma(s^0) > 0$. There is a cost to providing liquidity at date zero.$^{12}$

Along with Lemma 3, this implies the liquidity constraint holds with equality in period 0,

$$
\omega(s^0) = \phi_m + \phi_b. \tag{30}
$$

Setting initial liabilities above this minimum value, so that the liquidity constraint is slack in period 0, would require a higher initial tax rate and a correspondingly greater deadweight loss without any additional benefit.

The following proposition summarizes optimal policy.

$^{11}$Below, it is shown that the Friedman Rule is indeed optimal.

$^{12}$The planner, following the Friedman Rule, satiates liquidity demand, making the liquidity constraints slack for the consumers. But doing so is costly to the planner because it involves future tax liabilities.
Proposition 2: When prices are flexible and the government cannot actively trade an illiquid bond, optimal policy has the following characteristics: (1) the Friedman Rule holds, (2) the wage tax rate is non-decreasing for all states $s^t$, and increases in any state in which the liquidity constraint (28) would otherwise be violated, and (3) unanticipated price level jumps equate the real value of nominal liabilities to the present value of primary surpluses, thereby absorbing fiscal shocks.

We discuss each of these in turn. Proofs are in the online Appendix.

4.2 The Primacy of the Friedman Rule

The Ramsey Planner implements the Friedman Rule each and every period: liquidity demand is satiated and the seigniorage taxes are eliminated. In Section 3, liquidity demand could be satiated by buying illiquid bonds and providing cash and liquid bonds in return. Here, it will be shown that optimal policy satiates liquidity demand even though the illiquid bond is no longer available. Stated differently, optimal policy will satiate liquidity demand even if doing so requires changing fiscal policy.

4.3 Is Tax Smoothing Optimal?

The optimal path for the wage tax rate follows from (24) and (27). The Planner will keep the tax rate constant unless doing so would lead to a violation of the liquidity constraint and will change the tax rate in order to avoid violating that constraint. That is, tax smoothing is subordinated to the satiation of liquidity demand. If there is a conflict between the optimal provision of liquidity and smoothing of the tax rate, Friedman wins.

So, can there be conflicts between the Friedman Rule and maintaining a constant wage tax rate? In general the answer is, yes. The simplest way to see this is to consider a deterministic example in which unanticipated changes in the price level cannot play a role. Suppose government spending is constant, except for some known future period, $T$, when spending falls to zero for one period. If tax rates were constant, the Planner would set
the tax rate in period zero so that the present value of primary surpluses is equal to initial liabilities, \( \omega(s^0) = \phi_b + \phi_m \), in accordance with (30). In period T, when spending falls to zero, the government would run a surplus, reducing the liquidity supplied to the private sector. As shown in the online Appendix, this would create a liquidity shortage; that is, with a constant tax rate, \( \omega(s^T) < \phi_b + \phi_m \). But in our model, this liquidity shortage would violate the Friedman Rule; a constant tax rate cannot be optimal. Instead, optimal policy will set a low tax rate from period 0 to period T, and the resulting deficits will lead to growing liabilities and growing liquidity. This initial tax rate will be set low enough that there will be sufficient liabilities in period T to satiate liquidity demand despite that period’s surplus. At that point, the tax rate will be increased so that the present value of primary surpluses from then on is equal to outstanding liabilities, \( \phi_b + \phi_m \).\(^{13}\)

We next consider examples in which jumps in the price level play their familiar role as a fiscal shock absorber. These examples highlight the third principle of the conventional wisdom on debt management. For concreteness, we will focus on government spending shocks, and the potential conflict described above. In the first example, the distribution of the spending shocks is such that jumps in the price level might or might not accommodate the shocks without creating a liquidity shortage. In the second example, the distribution of the shocks will definitely result in a conflict, and the Planner will raise the tax rate when the conflict occurs. The tax rate will, however, retain some of the unit-root like characteristics associated with the literature following Barro: the tax rate will be held constant temporarily, but ultimately it will be raised.

4.3.1 Some Preliminaries:

The dynamics of the model follow from the model’s flow implementability condition (14) and the transversality condition (25), which is repeated here for convenience: \(^{13}\)The details can be found in Example 1 in the online Appendix.
\[ \omega(s^t) = \beta \sum_{s^{t+1}=s^t} \rho(s^{t+1}|s^t) \omega(s^{t+1}) + 1 - [n(s^t)]^2 \]  

(31)

\[ \lim_{n \to \infty} \beta^n \sum_{s^{t+1}=s^t} \rho(s^{t+n}|s^t) \omega(s^{t+n}) = 0. \]  

(32)

And of course the liquidity constraints \( \omega(s^t) \geq \phi_m + \phi_b \) must be satisfied since the Friedman Rule is optimal.

As shown in the online Appendix, under the Friedman Rule, \( 1 - [n(s^t)]^2 \) is equal to the primary surplus measured in terms of utility. So, (31) can be interpreted as the government’s flow budget constraint. We will denote the primary surplus by \( \sigma(s^t) \) when convenient.

Using (20), (21), (27), (17), and holding productivity constant \( (z(s^t) = 1) \),

\[ n(s^t) = \frac{1}{2} \left[ g(s^t) + \sqrt{[g(s^t)]^2 + 4 (1 - \tau_w(s^t))} \right] \]  

(33)

So, for a given tax rate, each realization of \( g(s^t) \) corresponds to a particular value of \( n(s^t) \).

In the examples that follow, \( g(s^t) \) is a sequence of independent draws from a distribution that has just two states, indexed by \( \{H, L\} \): government spending can take a low value with probability \( \rho_L \), or a high value with probability \( \rho_H \) \((= 1 - \rho_L)\).

And in each example, we will provisionally assume that \( \lambda \) (and \( \tau_w \)) is constant and ask if this assumption is consistent with a solution to the Ramsey Problem.

4.3.2 An example in which a conflict might not arise

This example shows that liquidity provision may, or may not, conflict with tax smoothing in a familiar setup: government spending can take either the low value, \( g_L \), or the high value, \( g_H \), in each period. For a given tax rate, employment – and therefore the surplus, \( \sigma = 1 - n^2 \) – can then take on one of two values,

\[ n(g_H, \tau_w) = \frac{1}{2} \left[ g_H + \sqrt{(g_H)^2 + 4 (1 - \tau_w)} \right] \]  

(34)
\[ n (g_L, \tau_w) = \frac{1}{2} \left[ g_L + \sqrt{(g_L)^2 + 4(1 - \tau_w)} \right] \] (35)

so \( n (g_H, \tau_w) > n (g_L, \tau_w) \) and \( \sigma (g_H, \tau_w) < \sigma (g_L, \tau_w) \). Corresponding to these are the stationary values for \( \omega (s^t) \) that satisfy (31)

\[ \omega (g_H, \tau_w) = \beta \left[ \rho_L \omega (g_L, \tau_w) + \rho_H \omega (g_H, \tau_w) \right] + \sigma (g_H, \tau_w) \] (36)
\[ \omega (g_L, \tau_w) = \beta \left[ \rho_L \omega (g_L, \tau_w) + \rho_H \omega (g_H, \tau_w) \right] + \sigma (g_L, \tau_w) \] (37)

Subtracting, we see that \( \omega (g_H, \tau_w) < \omega (g_L, \tau_w) \). As seen in Lemma 4, real liabilities in period zero will be set to \( \phi_m + \phi_b \). Iterating (31) forward, the tax rate is set to equate initial liabilities to the present value of primary surpluses.

\[ \omega (s^0) = \phi_m + \phi_b = \sigma (s^0) + \frac{\beta}{1 - \beta} (\rho_H \sigma (g_H, \tau_w) + \rho_L \sigma (g_L, \tau_w)) \] (38)

First, we suppose that spending at date zero is \( g_H \), and show there is no conflict between optimal liquidity provision and maintaining a constant tax rate. The government’s present value budget constraint gives a unique solution \( \tau_w^* \). The implied values of liabilities satisfy the liquidity constraints since \( \omega (g_L, \tau_w^*) > \omega (g_H, \tau_w^*) = \omega (s^0) = \phi_m + \phi_b \).

Next, we suppose that spending at date zero is \( g_L \) and show the constant tax rate solution will not be optimal in our model. Again, the Planner will set \( \omega (s^0) = \phi_m + \phi_b \). A solution with a constant tax rate would not satisfy the liquidity constraint when \( g_H \) is realized at some future date; a constant tax rate would imply that \( \omega (g_H, \tau_w) < \omega (g_L, \tau_w) = \omega (s^0) = \phi_m + \phi_b \). A constant tax rate will therefore not be consistent with the Ramsey solution; optimal policy will need to change the tax rate in order to prevent a violation of the liquidity constraint.

It is fairly straightforward to characterize optimal policy if \( g_L \) is realized in period zero. (24) and (27) imply that the tax rate will be constant unless the liquidity constraint would be violated. Should that occur, (i.e. should \( g_H \) be realized in some period) the Planner will
reoptimize, set the real value of liabilities to $\phi_m + \phi_b$ and increase the tax rate to satisfy the government’s present value budget constraint. More specifically, the Planner will set the tax rate in period zero to the value, $\bar{\tau}_w$, that satisfies the budget constraint:

$$
\phi_m + \phi_b = \sigma (g_L, \bar{\tau}_w) + \beta \{ \rho_H (\phi_m + \phi_b) + \rho_L \sigma (g_L, \bar{\tau}_w) \} \\
+ \beta^2 \{ \rho_L \rho_H (\phi_m + \phi_b) + (\rho_L)^2 \sigma (g_L, \bar{\tau}_w) \} + \ldots \\
= \sigma (g_L, \bar{\tau}_w) + \sum_{t=1}^{+\infty} (\beta \rho_L)^t \left\{ \frac{\rho_H}{\rho_L} (\phi_m + \phi_b) + \sigma (g_L, \bar{\tau}_w) \right\}
$$

(39)

The first term is the primary surplus in period 0. The remainder follows by iterating (31) forward. The first time $g_H$ is realized (with probability $\rho_H$), the Planner will reoptimize: the Planner will set liabilities to $\phi_m + \phi_b$ and increase the tax rate to $\bar{\tau}_w$, the same rate that would have been set had $g_H$ been realized in period 0. The solution from then on will be identical to the solution above when $g_H$ was realized in period 0.

Solving the present value budget constraint, (39),

$$
\sigma (g_L, \bar{\tau}_w) = (\phi_m + \phi_b) [1 - \beta] 
$$

(40)

So, if $g_L$ is realized in period zero, liabilities will be set to $\phi_m + \phi_b$. If $g_H$ is realized in period 1, as seen above, liabilities will be set to $\phi_m + \phi_b$. If, instead, $g_L$ is realized in period 1, (39) and (37) imply liabilities will also be $\phi_m + \phi_b$. Optimal policy therefore plans to set liabilities to $\phi_m + \phi_b$ regardless of the prevailing state in Period 1. The policy issues the minimum amount of liquidity to satisfy the constraint at date zero. This position is maintained with no change in the tax rate as long as the state continues to be $g_L$. When $g_H$ is realized for the first time, the Planner will change the tax rate to $\bar{\tau}_w$ and keep it constant thereafter. From

---

14 The problem is identical to that above when $g_H$ is realized in period 0. The Planner will set the real value of liabilities to the minimum value that satiates liquidity demand, $\phi_m + \phi_b$. A greater value of liabilities would incur additional costs without additional benefit.
\[ \phi_m + \phi_b = \sigma (g_H, \tau_w^*) + \frac{\beta}{1 - \beta} (\rho_H \sigma (g_H, \tau_w^*) + \rho_L \sigma (g_L, \tau_w^*)) < \frac{\sigma (g_L, \tau_w^*)}{1 - \beta} \]

Together with (40), this implies \( \sigma (g_H, \tau_w) < \sigma (g_L, \tau_w^*) \). Since the primary surplus is increasing in \( \tau_w, \tau_w^* > \tau_w \); so, the tax rate must increase when \( g_H \) is realized for the first time. The increase in the tax rate increases the present value of primary surpluses to \( \phi_m + \phi_b \) and the Planner adjusts the price level accordingly.

Why doesn’t the Planner smooth taxes and set the tax rate to the higher value, \( \tau_w^* \), initially? This would certainly be feasible, but it would result in a greater present value of future surpluses (and greater liquidity) prior to the first realization of \( g_H \). The liquidity constraint would be slack until \( g_H \) is realized so that the additional liquidity would have no additional benefit. But the higher tax rate would be costly. The Planner does not set the higher tax rate until it is required to provide satiation in liquidity demand.

**4.3.3 An example in which a conflict will eventually arise**

Here, we extend the example by considering a slightly different distribution of government spending. In this new example, a constant tax rate cannot be optimal.

As before, spending in period 1 may take the high value \( g_{H1} \) with probability \( \rho_H \) or the low value \( g_{L1} \) with probability \( \rho_L (= 1 - \rho_H) \). (Spending in period 0 will be \( g_{H1} \) to avoid a repetition of the issues already discussed in the previous example.) However, at some known date in the future, which we take to be period 2 for ease of exposition, there is a mean preserving spread in spending \( g (s^t) \). More specifically, the high and low values of spending change to \( g_{H2} \) and \( g_{L2} \), with the same probabilities \( \rho_H \) and \( \rho_L \). \( g_{H2} > g_{H1} \) and \( g_{L2} < g_{L1} \), while the expected value remains the same.

Once again, we provisionally assume that the wage tax rate is constant. The expected present value of primary surpluses pins down the value of \( \tau_w \). That is, iterating (31) forward and applying (32), the Planner chooses the tax rate to satisfy the period 0 present value
budget constraint

\[ \omega(s^0) = \phi_m + \phi_b = \sigma_{H1} + \beta (\rho_H \sigma_{H1} + \rho_L \sigma_{L1}) + \frac{\beta^2}{1 - \beta} (\rho_H \sigma_{H2} + \rho_L \sigma_{L2}) \]  

(41)

where \( \sigma_{Hi} = \sigma(g_{Hi}, \tau_w) \) and \( \sigma_{Li} = \sigma(g_{Li}, \tau_w) \) denote the primary surpluses for \( i = 1 \) and \( 2 \).

This policy is optimal if it keeps \( \omega(s^t) \geq \phi_m + \phi_b \) in future periods.

Now suppose it happens that government spending is high \((g_{H2})\) in period 2. In period 2, the present value budget constraint is

\[ \phi_m + \phi_b \leq \sigma_{H2} + \beta (\rho_H \sigma_{H2} + \rho_L \sigma_{L2}) + \frac{\beta^2}{1 - \beta} (\rho_H \sigma_{H2} + \rho_L \sigma_{L2}) \]  

(42)

For both (41) and (42) to hold, it must be that

\[ \sigma_{H1} + \beta (\rho_H \sigma_{H1} + \rho_L \sigma_{L1}) \leq \sigma_{H2} + \beta (\rho_H \sigma_{H2} + \rho_L \sigma_{L2}) \]

or that \( \sigma_{H1} \leq \sigma_{H2} \) since the expected surplus remains unchanged with a constant tax rate. But, \( g_{H2} > g_{H1} \); if the tax rate is held constant, then \( \sigma_{H1} > \sigma_{H2} \).

This is a contradiction. The Planner would respond to this higher realization of government spending in period 2 by adjusting the tax rate on labor income. The new tax rate would be determined so that (42) holds with equality.

As shown in the online Appendix, other changes in the distribution of shocks can also make a constant tax rate and liquidity provision incompatible. In particular, if the expected value of the primary surplus decreases (when the realizations of government spending become more extreme), a constant tax rate policy may not be optimal. The intuition is straightforward. If surpluses were on average larger early on, they would constitute a drain on liquidity, leaving insufficient liquidity after the distribution changed. Optimal policy would then set a lower tax rate early on and raise taxes when the realizations become more extreme.
4.4 Price Level Volatility

In stochastic flexible price models, unanticipated price level jumps play a central role in accommodating shocks. This third principle of the conventional wisdom of debt management is an essential part of optimal debt management in our model as well. In models where government bonds do not provide liquidity to the private sector, e.g. Chari et al. (1996), unexpected jumps in the price level make nominal liabilities state-contingent in real terms, and one obtains a Ramsey solution identical to the one obtained by Lucas and Stokey (1983) with state-contingent debt. When for example a shock increases government spending and the Planner holds tax rates constant, a deficit necessarily ensues. The present value of government surpluses falls and if nothing else changes, the intertemporal budget constraint is out of balance. However, the Planner can restore balance by simply increasing the price level to erode the real value of existing liabilities.

Unexpected jumps in the price level play the same role in our model, but the fact that government bonds provide liquidity imposes an additional constraint on optimal policy. The price level jump that absorbs fiscal shocks may create a liquidity shortage. That is, with spending exogenous and tax rates constant, there is no reason why the present value of primary surpluses cannot fall below $\phi_m + \phi_b$. In this case, a conflict arises between tax smoothing and liquidity provision. In the last section, the government could always avoid the liquidity shortage by buying private debt and paying for it with cash and/or bonds. This is not possible here, so something has to give should a conflict arise. When a conflict arises, liquidity provision trumps tax smoothing. Even though price flexibility makes nominal government bonds state contingent in real terms, one may not obtain the Lucas-Stokey solution, which would imply a constant tax rate with our preferences. This is the fundamental intuition for what was happening in our two examples.
4.5 Lessons for the Conventional Wisdom on Debt Management

When prices are flexible and the government does not have access to an illiquid bond, a hierarchy of the three principles of the conventional wisdom on debt management arises. The Friedman Rule reigns supreme; optimal policy will always satiate liquidity demand. Price level jumps play an important role in maintaining fiscal balance, but a conflict between liquidity provision and tax smoothing can arise. When a conflict arises, the first principle of the conventional debt management, satiating liquidity demand, trumps the second principle, maintaining constant tax rates.

The supremacy of the Friedman Rule follows from the optimality of uniform taxation. As shown in the last section, the Planner would avoid using the seigniorage taxes, setting instead a positive (and constant) labor tax rate in order to tax the three consumption goods equally. Here, the same uniform taxation motive leads the Planner to accept some tax rate variability in order to avoid using the seigniorage taxes.

In all of these examples, including the simple deterministic example, the Planner will hold the tax rate constant unless the realization of government spending in some period would result in too little liquidity. The Planner will avoid a liquidity shortage so that real liabilities will be sufficient to satiate liquidity demand. The Planner then reoptimizes by setting a new tax rate that equates the present value of expected future primary surpluses to the minimum required liquidity, $\phi_m + \phi_b$ in our model. The new tax rate will be held constant until another realization of government spending would result in insufficient liquidity. When that realization occurs, the Planner repeats the process. The tax rate will vary, but it will be constant between adjustments. In that sense some tax smoothing is optimal – the tax rate will be kept constant so long as doing so does not conflict with satiating liquidity.\(^{15}\)

\(^{15}\)As noted above, (24) implies that the tax rate will never decline. So although the tax rate will have some of the characteristics of a unit root process, it deviates from a unit root in an essential way.
5 Ramsey Solutions with Staggered Price Setting

This section adds monopolistic competition and Calvo-style staggered price setting; moreover, there is no indexing to steady state inflation.\textsuperscript{16} This creates a price dispersion that distorts household consumption decisions unless the Planner holds the aggregate price level constant, eschewing both the expected deflation associated with the Friedman Rule and the unanticipated jumps in the aggregate price level described in the last two sections.

In the last two sections, unanticipated jumps in the price level could be used to accommodate fiscal shocks. If these jumps are too costly to implement, then either taxes or deficit finance must be used instead, and the tradeoff between liquidity provision and tax smoothing is more difficult. Access to the third debt instrument eases this tradeoff. For example, a temporary increase in government spending can be financed by selling the illiquid bond, while still providing the level of money and bonds needed for liquidity; this allows the inevitable increase in the wage tax rate to be smoothed over time.

A substantial literature in the Barro tradition holds that short run tax distortions should be alleviated at the expense (or benefit) of a little long run consumption. According to this literature, in response to a temporary spending increase, the government should issue new liabilities to smooth the path of the tax hikes that will ultimately be needed. But, long run consumption would have to fall a little to service the higher level of debt; public debt and taxes would follow unit root processes as they move to efficiently absorb the fiscal shock. Here, it will be seen that this fundamental insight holds when the government has access to the illiquid debt instrument. When it does not, the Ramsey solutions are actually stationary.

Staggered price setting introduces a new state variable – the inflation rate – and it is difficult to obtain analytical results; we have to resort to numerical methods.\textsuperscript{17} There are only a few additional parameter settings to report in our simple model. $\beta$ is set so that the

\textsuperscript{16}The changes to the model are standard and there is no need to discuss them here; they are relegated to the online Appendix.

\textsuperscript{17}In particular, we solve the model for the Ramsey solution using \textit{Get Ramsey} (Levin and Lopez-Salido (2004)).
(annualized) steady state real rate of interest is 4%. The Calvo parameter is set so that the average price “contract” is four quarters. Finally, $\phi_m = \phi_b = \phi_c = 1/3$.

We will begin with a discussion of optimal taxation and inflation in the steady state. (Since firm prices are not indexed to steady state inflation, staggered price setting can also distort the steady state.) Then, we will show how the Ramsey Planner responds to shocks that have fiscal implications.

### 5.1 Optimal Tax Rates and Inflation in the Steady State

Table 1 reports the optimal steady state tax rates and inflation; these values are the same whether or not the government has access to the illiquid debt instrument.\(^{18}\) With staggered price setting, the Planner holds the aggregate price level virtually fixed, and the seigniorage taxes are positive. Calvo trumps Friedman in our model.

Table 1 shows that $I_c > I > 1$ in the case of staggered price setting. Why is this? Because near price stability is optimal, $I_c > 1$ and the three consumption goods will not be taxed uniformly. Cash good consumption will be taxed relative to credit good consumption. So the question is, how should the bond good be taxed? If $I$ were set equal to 1, (10) implies that the bond good - cash good margin would not be distorted, but (11) implies that this would come at the cost of an even greater distortion of the bond good - credit good margin. Similarly setting $I = I_c$ would leave the bond good - credit good margin undistorted, but this would come at the cost of a greater distortion of the bond good - cash good margin. Since utility is concave, optimal policy opts for two small distortions rather than one large one and spreads the consumption distortions by setting $I_c > I > 1$. So optimal policy does not satiate demand for liquidity in the steady state, and both seigniorage taxes, along with the wage tax, are used to finance expenditures.

In standard models without a liquidity role for government bonds, the Ramsey problem does not determine a steady state value for those bonds. A steady state value must be

\(^{18}\)The illiquid debt instrument can be used to affect tax rate dynamics by absorbing fiscal shocks; it plays no role in the steady state.
imposed when those models are used to consider optimal policy. In our model, on the other hand, the liquidity services of government bonds results in a well-defined steady state. Our model offers a complete theory of the optimal level of debt.

5.2 Optimal Response to Shocks with Fiscal Implications

Figure 1 shows the Ramsey Planner’s response to an increase in government spending that lasts just one period. We begin with the case in which the government has access to the illiquid debt instrument, depicted with the solid lines. Changes in the price level are costly, and the Planner makes very little use of them. Instead, the Planner makes some use of the seigniorage taxes, $I(s^t) - 1$ and $I^c(s^t) - I(s^t)$, but the big story is that the Planner issues new liabilities. This allows the Planner to avoid a larger, temporary wage tax hike. The new tax burden represents a negative wealth effect that forces the Planner to lower consumption and increase the work effort (not pictured in Figure 1). The important insight here is that the Planner can do all of this without compromising liquidity provision, since the government has access to the illiquid debt instrument $b_g(s^t)$. More precisely, the Planner can create any level of gross liabilities, $m(s^t) + b(s^t)$, by buying or selling $b_g(s^t)$; then, ordinary open market operations can create the right mix between cash and bond balances. In the present example, consumption falls; so, transactions balances $m(s^t) + b(s^t)$ can fall. The Planner sells $b_g(s^t)$ to buy back gross liabilities and to finance the deficit; as shown in Figure 1, net liabilities, $m(s^t) + b(s^t) - b_g(s^t)$, rise considerably.

After a few periods, the wage tax levels out at a permanently higher rate, collecting the new revenue that is needed to service the increase in net liabilities. And after a few periods, both consumption and work (not shown) assume permanently lower paths, reflecting the permanently higher wage tax rate. The fundamental insight attributed to Barro is preserved; net liabilities, consumption, and the wage tax rate exhibit unit root behavior.

When the government lacks access to an illiquid debt instrument, the optimal response to a temporary increase in government spending is quite different, as illustrated by the
dashed lines in Figure 1. The Planner cannot use $b_g(s^t)$ to separate liquidity provision from deficit finance, and the Planner does not issue more liabilities to avoid a big tax hike. Using deficit finance to smooth the wage tax would raise $m(s^t) + b(s^t)$, when consumption should go down. And since the cash and collateral constraints are binding, this would raise cash and bond good consumption; this distortion of consumption decisions would be too costly. Optimal debt management must trade off the benefits of wage tax smoothing with the costs of distorting consumption by providing additional liquidity. An immediate tax hike is optimal, and fluctuations in consumption and work are much larger than in the previous case.

Moreover, the Planner returns to the initial steady state solution. Why doesn't the Planner do just a little of what was optimal in the previous case? Suppose the Planner did, in fact, permanently raise liabilities to smooth tax hikes. The long run tax burden would rise, and total consumption would have to fall in the long run. As noted above, without the third debt instrument, liquidity must rise along with liabilities (since net and gross liabilities are the same in the present case). Since the cash and collateral constraints are always binding, cash and credit good consumption would rise as well. But, as already noted, total consumption must be permanently lower, and this would mean that credit good consumption would fall a lot; this is not optimal. Liabilities and the wage tax rate return to their original steady state values, and the unit root behavior is lost. In fact, only the fourth principle of conventional debt management remains.

So far, we have only considered government spending shocks, but other shocks can also have fiscal implications. Productivity, $z(s^t)$, is the only other shock term that has been modeled. An increase in government spending lowers the amount of output available for consumption; a decrease in productivity does the same thing. An increase in government spending requires debt financing if the wage tax rate is to be smoothed. A decrease in productivity lowers the real wage and thus wage tax revenue; it too requires debt financing if the the wage tax rate is to be smoothed. The Ramsey Planner’s response to a one period
decrease in productivity is remarkably similar (and therefore not reproduced) to the response pictured in Figure 1.

5.3 Implications for Conventional Wisdom on Debt Management

The introduction identified four principles that summarize the conventional wisdom on debt management. The first of these is the Friedman Rule. As is well known, the Friedman Rule is no longer optimal once staggered price setting and relative price distortions are introduced. In our model, nominal interest rates are positive, and there is a spread between the interest rate on government bonds and that on illiquid bonds issued by the private sector. Optimal policy does not satiate either the demand for cash or the demand for government bonds. The spread responds to shocks, and that response is greater when the government does not have access to the illiquid bond instrument. When the government can buy and sell the illiquid bond, it can reduce fluctuations in the marginal liquidity value of government debt and thereby reduce movements in the spread.

The second principle is that tax rates should be smoothed and government debt should fluctuate to absorb fiscal shocks. Tax smoothing remains a goal in our model when Calvo price setting is introduced, but the amount of smoothing that is optimal depends on the policy instruments that are available to the government. When the government has access to the illiquid bond instrument, the wage tax rate exhibits unit root behavior, rising permanently in response to a positive government spending shock. An increase in the government’s net liabilities facilitates this smoothing of the wage tax rate. In contrast, when the government cannot buy and sell the illiquid bond, the growth in the government’s net liabilities is also growth in its gross liabilities. The additional liquidity would distort consumption decisions; so, there is less tax smoothing, and the unit root behavior of the wage tax rate is lost.

The third and fourth principles refer to the relative merits of using unexpected inflation to tax nominal assets when prices are flexible, and achieving price stability when prices are sticky. In our simulations of the sticky price model, the liquidity of government bonds has
little effect on the optimality of price stability; similarly, the availability of the illiquid debt instrument has little effect. Optimal inflation is virtually zero in the steady state, and in the response to shocks with fiscal implications.

6 Conclusion

In this paper, we have extended a standard model – with cash goods, credit goods, and a distortionary wage tax – to allow for the fact that government bonds provide liquidity services. The implications for the four principles of conventional debt management (outlined in the introduction) have already been summarized at the ends of sections. Here, we simply note that those implications depend upon whether or not the government has access to an illiquid debt instrument, an instrument that the government can buy or sell without having any direct effect on a household’s liquidity or its consumption decisions.

How should one think about the illiquid debt instrument in our model? Here, it is modeled as government holdings of private sector bonds, which by our assumptions cannot be used as collateral, and therefore do not provide liquidity services. One possibility is to consider the illiquid bond an analytical device that highlights why liquidity provision and tax smoothing may come into conflict. It would not seem that governments currently utilize an illiquid debt instrument, and they certainly do not use private sector debt in the way our Ramsey Planner would. And if they did actively trade private sector debt, that debt might take on some of the liquidity characteristics of government debt. Governments, of course, can have sovereign wealth funds and non-liquid assets such as parks, oil rights, grazing rights, spectrum rights, and state owned enterprises. However these assets would be hard to use to finance short run deficits without creating disruptions in the real economy.

Another possibility is to think of short-term government debt as liquid and take long term government bonds as the illiquid debt instrument that is needed here, as in Greenwood et al. (2015). But some long term debt, especially the United States Treasury’s ten-year
note, is actively used in financial markets and provides significant liquidity services. A third possibility is to consider recent central bank purchases of private sector debt – such as asset backed securities – as an example of government holdings of illiquid claims on the private sector. Central banks appear, however, to be concerned about the potential for disruptions in private markets should these assets be sold, suggesting that these assets may not be bought and sold without any direct impact on the economy.

References


Table 1: Steady State Tax Rates and Inflation (annual rates)

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Figure 1: government spending shock

- Solid lines: government has access to illiquid bond
- Dashed lines: no access