Optimal Money and Debt Management: 
liquidity provision vs tax smoothing*

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Abstract

We extend a standard cash and credit good model by assuming that government bonds may be used as collateral in purchasing some kinds of consumption goods. In the extended model, the government cannot use its liabilities to finance deficits without affecting the amount of liquidity available to support consumption. The fundamental insights of Friedman and Barro survive largely intact if the government has an additional debt instrument that is “illiquid” – in the sense that its outstanding stock has no direct effect on consumption allocations. Absent such an instrument, optimal debt management, or provision of liquidity, may conflict with tax smoothing.

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1 Introduction

Government debt provides transactions balances that are needed by the private sector. But, the government must also finance ongoing public expenditures and cope with macroeconomic shocks that have fiscal implications. Optimal debt management must trade off the need for liquidity with these other constraints on fiscal policy. Two well-known principles provide the conventional wisdom on debt management. Friedman (1960), abstracting from fiscal considerations, argued for an extreme form of debt management: Demand for liquidity should be satiated by implementing the celebrated Friedman Rule; seigniorage tax revenues are eliminated. Barro (1979), abstracting from liquidity provision, argued that debt should be used to smooth distortionary tax rates in the short run, albeit at the expense (or benefit) of some long-run consumption; fiscal shocks cause consumption and public debt to follow unit root processes.\footnote{Aiyagari et al. (2002) formalize the Barro insight in a Ramsey Problem. They discuss when the unit root process emerges in a model with real bonds, no need for transactions balances, and exogenous limits on government assets. They find, among other things, that when the latter are not binding, the Planner will accumulate a war chest of assets so as not to have to use the distortionary tax in the future.}

This conventional wisdom finds support in standard models. Chari et al. (1996) – using a stochastic framework with cash and credit goods – have shown that the Friedman Rule is optimal even when a distortionary wage tax is the alternative to seigniorage taxes.\footnote{The principle of uniform taxation favors the use of the wage tax over the seigniorage tax to finance ongoing government expenditures; we will discuss this principle further in Section 2.} In addition, they show the wage tax rate can be smoothed; fiscal shocks are accommodated by unanticipated jumps in the price level, which represent a non-distortionary tax on existing nominal government liabilities. These unanticipated jumps in the price level, and the ongoing deflation implied by the Friedman Rule, are costless when prices are flexible. But when monopolistic competition and price rigidities are added, fluctuations in the aggregate price level bring a dispersion of individual firm prices that distorts household consumption decisions. Benigno and Woodford (2003), Schmitt-Grohe and Uribe (2004) and Siu (2004) have shown that the benefits of price stability trump Friedman in such a setting.
In this paper, we ask which of these results survive once we recognize that both money and
government bonds provide some degree of liquidity services. Standard analyses assume that
the government has a debt instrument – namely, a government bond – that is “illiquid” in
the sense that its supply does not directly affect needed transactions balances, and therefore
consumption allocations. For example, in the Friedman tradition, open market operations
can be used to satiate the demand for money; transactions balances are not limited by the
decrease in the supply of government bonds. Or in the Barro tradition, a temporary increase
in government spending can be bond financed to smooth the path of the tax hikes that will
ultimately be needed to service the increase in public debt; again, transactions balances and
consumption allocations are not affected by the path of government bonds. Once we admit
that government bonds also provide needed transactions balances, optimal debt management
may conflict with the smoothing of distortionary taxes.

To investigate these issues, we extend the well known model with cash and credit goods
in a natural way to allow for the liquidity of government bonds. We assume that there is
a third consumption good – a bond good – that can be purchased by posting government
bonds as collateral; the bond good, like the credit good, is actually paid for in the period
that follows. Both money and government bonds provide needed transactions balances, and
they are in this sense “liquid.” We will assume that private sector bonds cannot be used as
collateral, and are in this sense “illiquid” even though they can be readily bought and sold.³

We will show that if the government has access to a third debt instrument, an illiquid
instrument that does not provide liquidity services, then standard results survive largely
intact; open market swaps between liquid and illiquid debt can remove conflicts between
liquidity management and tax smoothing.⁴ In our model with flexible prices, the Ramsey
Planner implements the Friedman Rule; and the Planner holds the wage tax rate constant
over time and across all states of the economy, using unanticipated jumps in the price level

³Holmström and Tirole (1998) develop a model in which the private sector does not provide enough
liquidity, and the liquidity services of government bonds are welfare improving. Our simple model assumes
that only government bonds provide liquidity.

⁴We reviewed these standard results extensively in Canzoneri et al. (2010).
to accommodate fiscal shocks. With staggered price setting the Friedman Rule is no longer optimal but wage tax rates are smoothed in the short run and net government liabilities and consumption exhibit unit root behavior.

When the government does not have access to this third debt instrument, complications arise. With flexible prices, we will show that the Planner still implements the Friedman Rule, but the wage tax rate may fluctuate in certain states of the economy; it depends on the processes generating government purchases and productivity. We will provide an example in which the tax rate is held constant (at least, after the initial period), and other examples in which optimal debt management conflicts with a constant wage tax rate. With staggered price setting, the short run tax smoothing envisioned by Barro is gone, and the Ramsey solutions are stationary.

In this paper, we characterize the third, illiquid debt instrument as a claim on the private sector (or in our simple model, a risk free bond) that cannot be posted as collateral for the bond good. In what we will call the Unconstrained Model, the government can buy and sell this claim on the private sector; in the Constrained Model, we rule that out. We will discuss possible interpretations of this third debt instrument in the conclusion.

Of more technical interest is the fact that, in the Constrained Model, the Ramsey Planner has a meaningful problem in period zero. The Planner will not be tempted to simply inflate away existing nominal liabilities since this would deprive households of needed transactions balances in period zero. We will discuss the period - zero problem in some detail below.

Before we proceed to the analysis, we should discuss our assertion that government bonds provide liquidity services. The basic premise should not be controversial. U.S. Treasuries facilitate transactions in a number of ways: they serve as collateral in many financial markets, banks hold them to manage the liquidity of their portfolios, individuals hold them in money market accounts that offer checking services, and importers and exporters use them as transactions balances (since so much trade is invoiced in dollars). The empirical literature finds a liquidity premium on government debt, and moreover the size of that premium
depends upon the quantity of debt.\footnote{Empirical contributions to this literature include: Friedman and Kuttner (1998), Greenwood and Vayanos (2014), and Krishnamurthy and Vissing-Jorgensen (2012).}

The paper proceeds as follows: In Section 2, we present the basic model, and we show how various taxes distort household decisions. In section 3, we discuss general aspects of the Ramsey Problem with flexible prices. In Section 4, we derive the Ramsey solutions for the Unconstrained Model with flexible prices, and in Section 5, we characterize the Ramsey solutions in the Constrained Model with flexible prices.\footnote{We provide a more detailed derivation of the Ramsey solutions in the Appendix.} In Section 6, we show how the results change when we add monopolistic competition and staggered price setting. Finally, in Section 7, we conclude with a discussion of possible interpretations of the third, illiquid debt instrument.

\section{A Model with Liquid Government Bonds}

The basic structure of our model is easily explained. There are three consumption goods: a cash good, $c_m$, a bond good, $c_b$, and a credit good, $c_c$. Households face a cash in advance constraint for the cash good and a collateral constraint for the bond good. Government bonds are “liquid” in the sense that they can serve as collateral for the bond good. Households pay for their credit goods and bond goods at the beginning of the period that follows. Firms produce a perishable final product, $y$, which can be sold as a cash good, a bond good, or a credit good. Government purchases, $g$, are assumed to be credit goods. Consolidating the treasury and the central bank, the government has four taxes at its disposal: a labor tax, $\tau_w$, a profits tax, $\tau_Y$, and two seigniorage taxes, one on money and the other on bonds. All of the taxes are distortionary except for the profits tax.

In the Unconstrained Model, the government is able to lend to the private sector; that is, it can hold bonds issued by the private sector. These private sector bonds are “illiquid” in the sense that they cannot be used as collateral in purchasing the bond good. In the Constrained Model, the government does not have access to an illiquid debt instrument;
that is, the government is not allowed to buy or sell private sector bonds, even though (as we shall see) it would be welfare improving for it to do so.

We will begin by assuming that firms are perfectly competitive and prices are flexible. However, in the penultimate section, we will consider the implications of adding price rigidities. More precisely, we assume monopolistic competition and staggered price setting.

### 2.1 Structure of the Model

In each period \( t \), one of a finite number of events, \( s_t \), occurs. \( s^t \) denotes the history of events, \( (s_0, s_1, ..., s_t) \), up until period \( t \). The initial realization, \( s_0 \), is given. The probability of the occurrence of state \( s^t \) is \( \rho(s^t) \).

Competitive firms use a technology, which is linear in labor and has an aggregate productivity shock, \( z(s^t) \). Their goods are then sold to households (as cash goods, bond goods, or credit goods), and to the government (as credit goods). The feasibility constraint is then

\[
y(s^t) = c_m(s^t) + c_b(s^t) + c_c(s^t) + g(s^t)
\]  

Government purchases are assumed to follow an exogenous process.

The utility of the representative household is

\[
U = \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \rho(s^t) u [c_m(s^t), c_b(s^t), c_c(s^t), n(s^t)]
\]

\[
= E \sum_{t=0}^{\infty} \beta^t u [c_{m,t}, c_{b,t}, c_{c,t}, n_t]
\]

where \( n_t \) is hours worked. As these equations suggest, we will suppress the notation for the state, \( s^t \), when we think that it will cause no confusion.

In period \( t \) (or more precisely, state \( s^t \)), households may purchase state contingent securities, \( x(s^{t+1}) \), that pay one dollar in state \( s^{t+1} \) and cost \( Q(s^{t+1} | s^t) \). Households can also construct a riskless bond, \( B^c \), by purchasing a portfolio of claims that costs one dollar and
pays a gross rate of return $I^c(s^t)$ in every future state $s^{t+1}$:

$$1 = I^c(s^t) \sum_{s^{t+1}|s^t} Q(s^{t+1}|s^t) \quad (3)$$

We will consider two risk free bonds, $B^c$ and $B$. $B^c$ is issued by the private sector and pays a gross return $I^c$, which we will refer to as the CCAPM rate. Since households are identical, there will be no private sector demand for $B^c$ in equilibrium. However, the government can hold private sector bonds in the Unconstrained Model; so, the net demand for these bonds will be $B^c_g(s^t)$, the government’s holding of $B^c$. $B$ is the bond issued by the government, and it pays a gross rate of return $I$. $B$ can be used as collateral in the purchase of bond goods; $B^c$ cannot. In the Constrained Model, $B^c_g = 0$ is a constraint on government behavior; that is, the government does not have access to an illiquid debt instrument.

Each period is divided into two exchanges. In the financial exchange, public and private agents do all of their transacting except for the actual buying and selling of the final product; purchases of the final good occur in the goods exchange that follows, subject to the cash and collateral constraints: $M_t \geq P_t c_{m,t}$ and $B_t \geq P_t c_{b,t}$, where $P_t$ is the price of the final product. Households come into the financial exchange in period $t$ with nominal wealth $A(s^t)$. Their budget constraint in the financial exchange is

$$M(s^t) + B(s^t) - B^c(s^t) + \sum_{s^{t+1}|s^t} Q(s^{t+1}|s^t) x(s^{t+1}) \leq A(s^t) \quad (4)$$

where $x(s^{t+1})$ is the number of dollar claims purchased for state $s^{t+1}$. The evolution of wealth is governed by

$$A(s^{t+1}) = I(s^t) B(s^t) - I^c(s^t) B^c(s^t) + x(s^{t+1}) + [M(s^t) - P(s^t) c_{m}(s^t)] - P(s^t) [c_h(s^t) + c_c(s^t)] + [1 - \tau_w(s^t)] W(s^t) n(s^t) + [1 - \tau_{\Upsilon}(s^t)] \Upsilon(s^t) \quad (5)$$

where $\Upsilon$ is profits.
2.2 Household Optimization Problem

The household optimization problem is:

\[
V[A(s^t)] = \max\{u[c_m(s^t), c_b(s^t), c_c(s^t), n(s^t)] + \lambda^m(s^t) [M(s^t) - P(s^t) c_m(s^t)] + \lambda^b(s^t) [B(s^t) - P(s^t) c_b(s^t)] + \lambda^A(s^t) [A(s^t) - M(s^t) - B(s^t) + B^c(s^t) - \sum_{s^{t+1}|s^t} Q(s^{t+1}|s^t) z(s^{t+1})] + \beta \sum_{s^{t+1}|s^t} \rho(s^{t+1}|s^t) V[A(s^{t+1})]\}
\]

First order conditions give the pricing equation for state contingent claims

\[
Q(s^{t+1}|s^t) = \beta \rho(s^{t+1}|s^t) \left[ \frac{u_m(s^{t+1})}{u_m(s^t)} \frac{P(s^t)}{P(s^{t+1})} \right]
\]

Recalling that the gross nominal rate on CCAPM bonds, \( B^c \), is defined by (3), we arrive at the standard Euler equation

\[
1 = I_c^t \beta E_t \left[ \frac{u_{m,t+1}}{u_{m,t}} \frac{P_t}{P_{t+1}} \right]
\]

2.3 Tax Distortions

Since the final good can be sold as a cash good, a bond good, or a credit good, the marginal rate of transformation between these goods is one for one. Efficiency requires that the marginal rates of substitution in consumption are also equal to one.

The household’s first order conditions imply

\[
u_{m,t} = I_c^t u_{c,t}
\]

If the household gives up one dollar’s worth of the cash good, it can spend \( I_c^t \) dollars on the credit good, because credit goods avoid the cash in advance constraint. The first order
conditions also imply that the cash in advance constraint is binding (or \( \lambda^m_t > 0 \)) if \( I^c_t > 1 \). In this case, the household buys too few cash goods. \( I^c_t - 1 \) is the standard seigniorage tax on cash goods, and it is distortionary. Similarly, the first order conditions imply

\[
-u_{b,t} = (1 + I^c_t - I_t)u_{c,t}
\]

(10)

If the household gives up one dollar's worth of the bond good, it can spend \( 1 + I^c_t - I_t \) dollars on the credit good, because credit goods avoid the collateral constraint. The first order conditions also imply that the collateral constraint is binding (or \( \lambda^b_t > 0 \)) if \( I^c_t - I_t > 0 \). In this case, the household buys too few bond goods. \( I^c_t - I_t \) is a new seigniorage tax on bond goods, and again this tax is distortionary. An extended Friedman Rule – \( I^c_t = I_t = 1 \) – would eliminate both of these taxes on transactions balances.

The wage tax, \( \tau_{w,t} \), is also distortionary. First order conditions imply

\[
-u_{n,t} = (1 - \tau_{w,t}) \left( \frac{W_t}{P_t} \right) u_{c,t} = \left( \frac{1 - \tau_{w,t}}{I^c_t} \right) \left( \frac{W_t}{P_t} \right) u_{m,t} = \left( \frac{1 - \tau_{w,t}}{1 + (I^c_t - I_t)} \right) \left( \frac{W_t}{P_t} \right) u_{b,t}
\]

(11)

Note that \( I^c_t > 1 \) and \( I^c_t - I_t > 1 \) distort the consumption - leisure margins in addition to the relative consumption margins. By contrast, the wage tax affects all three consumption goods equally. If the consumption - leisure margins should be taxed, then the wage tax should be used to do it. This reflects the Atkinson and Stiglitz (1976) uniform taxation principle, as Chari et al. (1996) have noted for preferences that are homothetic and separable in consumption and leisure. This fact figures prominently in the results that follow.

In what follows, we will work with the utility function

\[
u (c_{m,t}, c_{b,t}, c_{c,t}, n_t) = \phi_m \log (c_{m,t}) + \phi_b \log (c_{b,t}) + \phi_c \log (c_{c,t}) - \frac{1}{2} n_t^2, \quad (12)\]


although our results readily extend to more general preferences. This functional form facilitates our exposition: as we shall see, it makes the optimal wage tax constant over time and across states in the flexible price version of our Unconstrained Model. So, we can interpret movements in that tax rate in our other models as exhibiting a tradeoff between tax smoothing and the Friedman Rule.

3 The Ramsey Problem with Flexible Prices

We can solve the Ramsey Problem analytically when prices are flexible. We will follow a primal approach: first, we derive a sequence of flow implementability conditions by using household first order conditions to eliminate relative prices and tax rates in the household flow budget constraints; there are also implementability conditions associated with various inequality constraints, and a feasibility condition. Then, we choose the quantities of consumption and labor that optimize household utility subject to the relevant constraints. We also choose the quantities of real government liabilities, \( m(s^t) \) and \( b(s^t) \); since there are inequality constraints on the consumption of cash and bond goods, we imbed some complementary slackness conditions in the implementability conditions, allowing for the possibility that the constraints are not binding. These aspects of the Ramsey problem are common to the Ramsey solution of both the Constrained Model and the Unconstrained Model, but the Planner has an extra policy instrument, \( b_c^g(s^t) \), in the latter.

In this section, we discuss aspects of the problem that are common to both models. In Section 4, we derive the Ramsey Solutions for the Unconstrained Model, and in Section 5, we characterize the Ramsey solutions for the Constrained Model.

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7In particular, as explained in the Appendix, our derivation of the Ramsey solution extends to preferences that are homothetic over the three consumption goods and weakly separable across employment and consumption.
3.1 Implementability Conditions and the Feasibility Condition

The first step is to derive the flow implementability conditions. Using (5) to eliminate \( x(s^{t+1}) \), and recalling the definition of \( I^c(s^t) \), the household budget constraint (4) can be written as

\[
A(s^t) = \sum_{s^{t+1}|s^t} [Q(s^{t+1}|s^t) A(s^{t+1})] + M(s^t) \left[ 1 - \frac{1}{I^c(s^t)} \right] + B(s^t) \left[ 1 - \frac{I(s^t)}{I^c(s^t)} \right]
\]

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\]

Next, we imbed the complementary slackness conditions for \( m(s^t) \) and \( b(s^t) \). The cash in advance constraint is binding for \( I^c(s^t) > 1 \), and we have \( I^c(s^t) = 1 \) if this constraint is not binding. So for all \( I^c(s^t) \geq 1 \), we have

\[
M(s^t) \left[ 1 - \frac{1}{I^c(s^t)} \right] = P(s^t) c_m(s^t) \left[ 1 - \frac{1}{I^c(s^t)} \right]
\]

Similarly, the collateral constraint is binding if \( I^c(s^t) > I(s^t) \), and we have \( I^c(s^t) = I(s^t) \) if this constraint is not binding. So, for all \( I(s^t) \) satisfying \( 1 \leq I(s^t) \leq I^c(s^t) \), we have

\[
B(s^t) \left[ 1 - \frac{I(s^t)}{I^c(s^t)} \right] = P(s^t) c_b(s^t) \left[ 1 - \frac{I(s^t)}{I^c(s^t)} \right] = P(s^t) c_b(s^t) \left[ \frac{I^c(s^t) - I(s^t)}{I^c(s^t)} \right]
\]

Substituting the complementary slackness conditions – (14) and (15) – into (13), and using household first order conditions to eliminate relative prices and \( Q(s^{t+1}|s^t) \), we arrive at the flow implementability conditions

\[
\omega(s^t) = \beta \sum_{s^{t+1}|s^t} \rho [s^{t+1}|s^t] \omega(s^{t+1}) + 1 - [n(s^t)]^2
\]

where

\[8\] The Appendix provides a more detailed account of the derivations in this subsection.
\[ \omega(s^t) \equiv \left[ \frac{\phi_m}{c_m(s^t)} \right] \left[ \frac{A(s^t)}{P(s^t)} \right] = \left[ \frac{\phi_m}{c_m(s^t)} \right] \left[ m(s^t) + b(s^t) - b^c(s^t) \right] \]

In the Appendix, we show that \( 1 - [n(s^t)]^2 \) is equal to the primary surplus measured in terms of utility. So, (16) can be interpreted as the government’s flow budget constraint.

We also have the implementability constraints implied by \( 1 \leq I(s^t) \leq I^c(s^t) \); from equations (9) and (10),

\[ \frac{c_{c,t}}{\phi_c} \geq \frac{c_{b,t}}{\phi_b} \geq \frac{c_{m,t}}{\phi_m} \] (17)

And finally, the feasibility constraint when prices are flexible is

\[ c_m(s^t) + c_b(s^t) + c_c(s^t) + g(s^t) = z(s^t) n(s^t) \] (18)

### 3.2 The Ramsey Planner’s First Order Conditions

The Ramsey Planner maximizes household utility subject to the sequence of implementability constraints (16) (with Lagrange multipliers \( \lambda(s^t) \)), the implementability constraints (17) (with multipliers \( \eta^c(s^t) \) and \( \eta^m(s^t) \) respectively), the feasibility conditions (18) (with multipliers \( \mu(s^t) \)), and finally, the cash and collateral constraints themselves (with multipliers \( \gamma^m(s^t) \) and \( \gamma^b(s^t) \) respectively). We have imbedded the complementary slackness conditions in the implementability conditions; so, they are valid whether or not the cash and collateral constraints are binding.

The Planner’s first order conditions for periods \( t \geq 0 \) are:
\[ c_c(s^t) : \quad \frac{\phi_c}{c_c(s^t)} = \mu(s^t) - \frac{\eta^c(s^t)}{\phi_c} \]  
(19)

\[ c_b(s^t) : \quad \frac{\phi_b}{c_b(s^t)} = \mu(s^t) + \gamma^b(s^t) + \frac{\eta^c(s^t) - \eta^m(s^t)}{\phi_b} \]  
(20)

\[ c_m(s^t) : \quad \frac{\phi_m}{c_m(s^t)} = \mu(s^t) + \gamma^m(s^t) \left[ 1 - \frac{\omega(s^t)}{\phi_m} \right] + \frac{\eta^m(s^t)}{\phi_m} \]  
(21)

\[ n(s^t) : \quad [1 + 2\lambda(s^t)] n(s^t) = \mu(s^t) z(s^t) \]  
(22)

\[ b(s^t) : \quad \gamma^m(s^t) = \gamma^b(s^t) (\equiv \gamma(s^t)) \]  
(23)

\[ \omega(s^{t+1}) : \quad \lambda(s^{t+1}) - \lambda(s^t) = \gamma(s^{t+1}) \frac{c_m(s^{t+1})}{\phi_m} \]  
(24)

The last two conditions describe optimal money and debt management. The choice of \( \omega \equiv (m + b - b_g^c)(\phi_m / c_m) \) sets the level of net government liabilities; then the choice of \( b \) (and \( b_g^c \) in the Unconstrained Model) sets its composition. We will describe the first order condition for \( b_g^c \) in Section 4.

The optimality condition for \( b \), (23), says that the Planner’s shadow prices for satisfying the cash and collateral constraints, \( \gamma^m(s^t) \) and \( \gamma^b(s^t) \), must be equal. Why? For any level of net liabilities, open market operations can change the composition of money and bonds to equalize the cost of these constraints.

The Lagrange multiplier on the implementability constraint, \( \lambda(s^t) \), represents the shadow price of the debt burden, and as we shall see, \( \tau_w(s^t) \) is an increasing function of \( \lambda(s^t) \). If the liquidity constraints are binding (\( \gamma(s^t) > 0 \)), then (24) implies that \( \lambda(s^t) - \lambda(s^{t-1}) > 0 \); the Ramsey Planner issues more real liabilities and sets a higher \( \tau_w(s^t) \) to help finance the increased tax burden. If the inequality constraints are never binding (\( \gamma(s^t) = 0 \)), then \( \lambda(s^t) = \lambda \), and \( \tau_w(s^t) \) is constant over time and across states.
4 The Unconstrained Model with Flexible Prices

In the Unconstrained Model, the Planner also has access to the illiquid debt instrument \( b^c_g(s^t) \). The advantage of this policy instrument is that, unlike money and government bonds, it can be changed without having any direct effect on the transactions balances necessary to purchase the cash and bond goods.

4.1 Primacy of the Friedman Rule

As shown in the Appendix, \( \eta^c(s^t) \) and \( \eta^m(s^t) \) are always equal to zero in the Unconstrained Model; the inequality constraints (17) are not binding. Since \( \gamma(s^t) \) is also equal to zero, the first order conditions (19), (20) and (21) imply that

\[
\frac{\phi_m}{c_m(s^t)} = \frac{\phi_b}{c_b(s^t)} = \frac{\phi_c}{c_c(s^t)} \tag{25}
\]

And from the households’ first order conditions, (9) and (10), we must have

\[
I^c(s^t) = I(s^t) = 1 \tag{26}
\]

So, the Ramsey planner implements the Friedman Rule in every period: the seigniorage taxes are eliminated. Is the Friedman Rule consistent with smoothing the wage tax rate? This is the question to be studied in the remainder of the section.

4.2 The Period Zero Problem

The natural state variable in our setup is \( A(s^t) = M(s^t) + B(s^t) - B^c_g(s^t) \), the net nominal liabilities of the government; this suggests that the initial condition should be placed on \( A(s^0) \). But, doing so would lead to a familiar problem in our Unconstrained Model: the Planner would be tempted to simply inflate away its old debt to the private sector, and then inject new transactions balances in exchange for \( B^c_g(s^0) \).
There is really no meaningful period-zero optimization in the Unconstrained Model, and thus there is no theory of the optimal level of government liabilities for this model.\textsuperscript{9} This problem is well understood in the literature. To set aside these issues, we will modify the Ramsey problem at date zero. Specifically, we will let the Ramsey planner set the real values \( m(s^0), b(s^0), \) and \( b^c_g(s^0), \) subject to a given (possibly zero) value for real net government liabilities, \( m(s^0) + b(s^0) - b^c_g(s^0). \)

Note that the Planner can satiate the households’ need for liquidity in period zero by issuing public sector liabilities in exchange for private sector debt, \( b^c_g(s^0). \) It is perhaps no surprise that, as shown in the Appendix, the optimality condition for \( b^c_g(s^0) \) gives \( \gamma(s^0) = 0; \) the liquidity constraints are not binding in period zero. The rest of the period-zero solution will be discussed below.

### 4.3 The Stochastic Equilibria that Follow Period Zero

In fact, the first order condition for \( b^c_g(s^t) \) sets \( \gamma(s^t) = 0 \) in any state and at any time. The liquidity constraints are never binding. The intuition is that, once again, the Planner can always provide the necessary gross liabilities by exchanging \( b^c_g(s^t) \) for \( m(s^t) + b(s^t), \) and then the Planner can provide the required liquidity for the cash and collateral constraints individually by using the usual open market operations. And since \( \gamma(s^t) = 0, \) the first order condition for \( \omega(s^t) \) then implies that \( \lambda(s^t) = \lambda, \) a constant in all states and at any time.

If there are no revenues from the seigniorage taxes, what funds the initial liabilities, the average level of government spending, and adverse fiscal shocks? There are two other taxes: the wage tax, which is distortionary, and unanticipated jumps in the price level, which are not. As shown in the Appendix, the optimal tax rate is positive:

\[
\tau_w(s^t) = 1 - \frac{1}{1 + 2\lambda(s^t)} > 0
\]  

\textsuperscript{9}This is an example of the familiar problem of multiple steady states under optimal policy. Initial conditions will determine real liabilities.
Wage tax rates are high when the fiscal burden \( \lambda(s^t) \) is high. But since \( \lambda(s^t) = \lambda \), the tax rate is held constant in the Unconstrained Model; the distortion it creates is perfectly smoothed over time and across states. Why is the wage tax rate positive while the two seigniorage taxes are set to zero? As explained in section 2.3, using the seigniorage taxes would violate the principle of uniform taxation.

The wage tax rate is constant, and it is set at a level that will finance the initial government liabilities and average government expenditure. But how does the Planner cope with fiscal shocks? Suppose for example that there is an unexpected increase in government spending. As shown in the Appendix, hours worked increase when government spending rises, and this provides more revenue. The Planner can also engineer a jump in the price level that taxes existing nominal liabilities. Surprise inflation is a non-distortionary tax, and it has no cost in our flexible price models. This decreases liquidity, but consumption optimally falls in response to an increase in government spending. So, the mix of financing methods depends on the household’s consumption - leisure tradeoff.

The bottom line is that the Planner always implements the Friedman Rule, and it does not conflict with a constant wage tax rate; the Planner uses unanticipated jumps in the price level to accommodate shocks with fiscal implications. Simply put, the Unconstrained Model has two types of money, two liquidity constraints, and one illiquid debt instrument, \( b_g(s^t) \). It is a natural extension of the standard model (in which government bonds do not provide liquidity), and standard results are obtained.

5 The Constrained Model with Flexible Prices

In the Constrained Model, the government is not allowed to hold private sector debt; that is, \( b_g \equiv 0 \) and \( \omega \equiv (m + b) (\phi_m/c_m) \). When the government does not have an illiquid debt instrument, the felicitous outcome described in Section 4 may fall apart, and the Ramsey solutions become more interesting. Without the first order condition for \( b_g(s^t) \), we can no
longer conclude at the outset that the liquidity constraints will never be binding. And, among other things, this leads to a meaningful Planner’s problem in period zero.

It is convenient to redefine the shadow price of liquidity in the first order condition for \( \omega(s^t) \):

\[
\tilde{\gamma}(s^t) \equiv \gamma^m(s^t) \cdot \frac{c_m(s^t)}{\phi_m} = \lambda(s^t) - \lambda(s^{t-1})
\]

(28)

\( \tilde{\gamma}(s^t) \) defines the shadow price in units of the cash good.

5.1 Primacy of the Friedman Rule

Once again, as shown in the Appendix, the equalities (25) hold. So, the Ramsey planner implements the (extended) Friedman Rule in every period: the seigniorage taxes on transactions balances are eliminated. Is the Friedman Rule consistent with smoothing the wage tax rate in the Constrained Model? This is the question to be studied on the remainder of the section.

In the Appendix we show that when the Friedman Rule holds, the cash and collateral constraints are equivalent to the Planner’s liquidity constraint:

\[
\omega(s^t) \geq \phi_m + \phi_b.
\]

(29)

The constraint holds with equality when \( \gamma(s^t) > 0 \).

5.2 The Period Zero Problem

As noted above, the natural state variable in our model is \( A(s^t) = M(s^t) + B(s^t) \), the nominal value of government liabilities. In the Constrained Model, we will assume that the Ramsey Planner inherits some \( A(s^0) > 0 \), and the Planner’s choice of \( P(s^0) \) determines the real value of government liabilities in period zero. Here, the Planner will not be tempted to inflate away its initial liabilities. Households would not have the transactions balances they need in period zero. (In the last section, the Planner could inflate away its old debt, and
then inject new transactions balances in exchange for $B^c_g(s^0)$.

As noted above, the Planner’s choice of $P(s^0)$ determines the real value of nominal liabilities; then open market operations – the choice of $b(s^0)$ – determine the optimal mix of liquidity between money and bonds. The optimal choice of $\omega(s^0)$ implies that

$$\tilde{\gamma}(s^0) = \lambda(s^0) \quad (30)$$

There is a cost to providing liquidity at date zero. (30) equates the value of initial liquidity, $\tilde{\gamma}(s^0)$, to the shadow cost of the future taxes needed to service the initial liabilities, $\lambda(s^0)$; this cost is of course positive. And since $\tilde{\gamma}(s^0) > 0$, $\omega(s^0) = \phi_b + \phi_m$.

The rest of the Planner’s solution will become clear in what follows. But, before going on, we should note that the Constrained Model does offer a theory of the optimal level of public debt.

### 5.3 The Stochastic Equilibria that Follows Period Zero

We show in the Appendix that the Ramsey problem leads to a dynamical system consisting of (29) and (28),

$$\tilde{\gamma}(s^t) \left[ \omega(s^t) - (\phi_m + \phi_b) \right] = 0 \quad (31)$$

$$\omega(s^t) = \beta \sum_{s^{t+1}|s^t} \rho(s^{t+1}|s^t) \omega(s^{t+1}) + 1 - \left[ n(s^t) \right]^2 \quad (32)$$

$$n(s^t) = \frac{1}{2z(s^t)} \left[ g(s^t) + \sqrt{[g(s^t)]^2 + \frac{4[z(s^t)]^2}{1 + 2\lambda(s^t)}} \right] \quad (33)$$

$$\tau_w(s^t) = 1 - \frac{1}{1 + 2\lambda(s^t)} > 0 \quad (34)$$
and finally, the transversality condition

\[ \lim_{n \to \infty} \beta^n \sum_{s^{t+1} | s^t} \rho \left( s^{t+n} | s^t \right) \omega \left( s^{t+n} \right) = 0 \]  \hspace{1cm} (35)

This system of equations already incorporates the Friedman Rule.

The Planner’s liquidity constraint may never be binding; that is, \( \tilde{\gamma}(s^t) \) may be equal to 0 for all \( s^t \). In this case, (28) implies that \( \lambda(s^t) = \lambda \), and the Planner holds the wage tax rate constant. The Friedman Rule does not conflict with tax smoothing. On the other hand, if the liquidity constraint does become binding at some point (\( \tilde{\gamma}(s^t) > 0 \)), the Planner injects additional liquidity. Then, (28) implies that \( \lambda(s^t) > \lambda(s^{t-1}) \); the Friedman Rule is not consistent with a constant wage tax rate.

Optimal policy never involves a reduction in the tax rate. A path with a higher initial tax rate followed by a lower tax rate is inferior to a path in which the tax rate is smoothed and the initial tax rate is lower. The smoothed tax rate does not conflict with liquidity provision – the Friedman rule is maintained. The liquidity constraint may be slack in some states and that additional liquidity can serve as a cushion for states in which liquidity demand is higher.

So, can the liquidity constraint ever be binding? In the Unconstrained model, it could not. The Planner could always buy private sector bonds, and pay for them with new liabilities. Here that tactic is not allowed. In the Appendix, we show that two types of solution can satisfy the optimality conditions. In the first, \( \tilde{\gamma}(s^t) \) is zero after the initial period; the liquidity constraints are never binding after period zero. In the second, \( \tilde{\gamma}(s^t) \) is positive in some states after the initial period; the liquidity constraints are (perhaps only occasionally) binding. Next, we provide examples of each type of solution.
5.4 Example 1: $\tau_w(s^t)$ is constant after period zero.

In this example, we hold productivity constant, and we let $g(s^t)$ be a sequence of independent draws from a time-invariant distribution. There are just two states, indexed by $\{H, L\}$: $g(s^t)$ can take a low value $g_L$, with probability $\rho_L$, or a high value $g_H$, with probability $\rho_H$ ($= 1 - \rho_L$). We begin by characterizing the solution for periods $t \geq 1$; then, we turn to the problem in period zero to assure that the liquidity constraints are satisfied.

In the candidate solution, the liquidity constraints are never binding; so, (28) implies that $\lambda(s^t) = \lambda$, and the wage tax rate is constant. Then, (33) implies that hours worked can take only two values:

$$n_H = \frac{1}{2} \left[ g_H + \sqrt{(g_H)^2 + \frac{4}{1 + 2\lambda}} \right]$$  \hspace{1cm} (36)

$$n_L = \frac{1}{2} \left[ g_L + \sqrt{(g_L)^2 + \frac{4}{1 + 2\lambda}} \right]$$  \hspace{1cm} (37)

And, the implementability conditions (32) imply the corresponding values of $\omega_H$ and $\omega_L$:

$$\omega_H = \beta [\rho_L \omega_L + \rho_H \omega_H] + 1 - (n_H)^2$$  \hspace{1cm} (38)

$$\omega_L = \beta [\rho_L \omega_L + \rho_H \omega_H] + 1 - (n_L)^2$$  \hspace{1cm} (39)

Note that $\omega_H < \omega_L$; consumption is low when government spending is high, and government liabilities can therefore be low when government spending is high.

It remains to be shown that these values of $\omega_H$ and $\omega_L$ satisfy the liquidity constraints. As shown in Section 5.1, $\omega(s^0) = \phi_m + \phi_b$ no matter which state occurs in period zero; then, (32) implies that

$$\phi_m + \phi_b = \beta [\rho_L \omega_L + \rho_H \omega_H] + 1 - [n(s^0)]^2$$  \hspace{1cm} (40)

So, $n(s^0)$ cannot depend on the initial state either.
Suppose \( g(s^0) = g_H \), and the planner sets \( n(s^0) = n_H \); we get

\[
\omega_H = \omega(s^0) = \phi_m + \phi_b \tag{41}
\]

and since \( \omega_H < \omega_L \),

\[
\omega_L > \phi_m + \phi_b.
\]

So, the proposed values of \( \omega_H \) and \( \omega_L \) satisfy the liquidity constraint, and the solution with a constant tax rate is feasible when \( g(s^0) = g_H \).

Intuitively, the constant tax rate, \( \tau_w \), is set at a level that will collect enough revenue to finance the initial government liabilities and an “average” level of government spending. Spending is high in period zero, and the Planner finances the deficit by issuing new liabilities. If government spending remains high in period 1, the Planner finances the new deficit by issuing new liabilities; an unanticipated increase in the price level keeps \( \omega_H = \phi_m + \phi_b \). This process continues as long as government spending is high. When government spending is low, the government has a surplus and the Planner does just the opposite.

The initial period is a little more complicated when \( g(s^0) = g_L \). It must still be the case that \( n(s^0) = n_H \). As shown in the Appendix, the specified values of \( \omega_H \) and \( \omega_L \) still satisfy the liquidity constraints. But, in the initial period the Planner sets a lower wage tax rate. Starting in period 1, the tax rate rises but remains constant thereafter.

Summing up, in Example 1, the Friedman Rule is always optimal. But, after the initial period, this does not conflict with a constant wage tax rate. Fiscal shocks are absorbed by unanticipated jumps in the price level.

### 5.5 Example 2: \( \tau_w(s^t) \) is not constant after period zero

We again hold productivity constant, and we let the path of government spending be deterministic: \( g_T = 0 \) in some period \( T > 0 \), while \( g_t = g > 0 \) in all periods before and after period \( T \). Then, (28) and (34) imply that the wage tax rate cannot be constant.
Suppose to the contrary that the constraint is never binding, and therefore $\lambda_t = \lambda$ and the wage tax rate is constant; we will show that this leads to a contradiction. (33) implies

$$n_t = \begin{cases} n \equiv n(\lambda, g) = \frac{1}{2} \left[ g + \sqrt{(g)^2 + \frac{4}{1+2\lambda}} \right] & \text{for } t \neq T \\ n_T \equiv n(\lambda, 0) = \sqrt{\frac{1}{1+2\lambda}} & \text{for } t = T \end{cases}$$ (42)

Once again, $\omega_0 = \phi_m + \phi_b$. So, iterating (32) forward, and using the transversality condition (35), we have

$$\omega_0 = \phi_m + \phi_b = \frac{1 - n^2}{1 - \beta} + \beta^T (n^2 - n_T^2)$$ (43)

To see this, note that $(1 - n^2) / (1 - \beta)$ is what the present value would be if $n_T^2$ were also equal to $n^2$; it is not, and $\beta^T [(1 - n_T^2) - (1 - n^2)] = \beta^T (n^2 - n_T^2)$ is the correction factor.

The constant wage tax rate must be set so that the wage tax pays for the average level of government spending and the initial liabilities.

Before period $T$, tax revenues will be less than government spending, and the government must issue new liabilities to cover the deficit. But here, unlike example 1, the equilibrium is deterministic, and the Planner cannot use an unanticipated increase in the price level to keep $\omega = \phi_m + \phi_b$. Excess liabilities build up to

$$\omega_T - \omega_0 = T (1 - \beta) \beta^{T-1} (n^2 - n_T^2)$$ (44)

in period $T$. In that period, spending falls to zero; the wage tax base also falls, but tax revenue does not fall to zero. So, the government must run a surplus in period $T$; it has to retire debt by the amount of the surplus. If excess liabilities have not built up to the amount of the surplus, then the liquidity constraint becomes binding. And, the constant wage tax rate cannot be optimal. In the Appendix, we show that the constraint is binding in period $T$ for a wide range of parameter values. So, the Planner can not set a constant tax rate.

What is the optimal policy? We can show that the liquidity constraint is only binding
in period $T$. That is, the Planner can set

$$\lambda(s^t) = \begin{cases} 
\lambda_b & \text{for } t \leq T \\
\lambda_a & \text{for } t > T 
\end{cases}$$

After the constraint is hit in period $T$, the planner will reset liabilities to $\omega_{T+1} = \phi_m + \phi_b$. Iterating (32) starting at $T + 1$ and applying (35), we see that $\lambda_a$ is determined by

$$\phi_m + \phi_b = \frac{1}{1 - \beta} \left\{ 1 - [n(\lambda_a, g)]^2 \right\}.$$ 

Next, iterating (32) from date 0 to date $T + 1$, we see that $\lambda_b$ is determined by

$$\phi_m + \phi_b = \sum_{t=0}^{T} \beta^t \left\{ 1 - [n(\lambda_b, g)]^2 \right\} + \beta^T \left\{ [n(\lambda_b, g)]^2 - [n(\lambda_b, 0)]^2 \right\} + \beta^{T+1} (\phi_m + \phi_b)$$

where $\beta^T \left\{ [n(\lambda_b, g)]^2 - [n(\lambda_b, 0)]^2 \right\}$ is once again a correction factor.

The constant wage tax rates, before and after period $T$, are determined by (34). In the Appendix, we show that $\lambda_a > \lambda_b$; so, the tax rate after period $T$ must be higher than the tax rate before period $T$. This reflects the fact that after $T$ there will be no period is which government spending falls; higher “average” spending requires more tax revenues.

The bottom line is that the Friedman Rule, which is always optimal in the Constrained Model, is incompatible with a constant tax rate in this example.

### 5.6 Example 3: $\tau_w(s^t)$ may not be constant after period zero

Here, we extend example 1 and ask if the liquidity constraint will be binding when in some period $T > 1$, the distribution of government spending changes; if not, then a constant $\lambda$ and $\tau$ will be optimal. To keep the exposition simple, we will let $T = 2$. In period 1, spending may take the high value $g_{H1}$ with probability $\rho_H$ or the low value $g_{L1}$ with probability $\rho_L(= 1 - \rho_H)$. Starting in period 2, the high and low values change to $g_{H2}$.
and \( g_{L2} \), but the probabilities \( \rho_H \) and \( \rho_L \) remain the same.\(^{10}\)

Iterating (32) forward and using (35), the Ramsey policy must satisfy

\[
\phi_m + \phi_b = 1 - (n_{H1})^2 + \beta \{ \rho_H [1 - (n_{H1})^2] + \rho_L [1 - (n_{L1})^2] \} \\
+ \frac{\beta^2}{1 - \beta} \{ \rho_H [1 - (n_{H2})^2] + \rho_L [1 - (n_{L2})^2] \}
\]

(45)

where \( n_{Hi} = n(g_{Hi}, \lambda) \) and \( n_{Li} = n(g_{Li}, \lambda) \) for \( i = 1 \) and \( 2 \). The expected present value of primary surpluses (measured in terms of utility) pins down the value of \( \lambda \).

This policy is feasible if it keeps \( \omega(s') \geq \phi_m + \phi_b \). In the Appendix, we show the constraint for period 1 is

\[
\rho_H [1 - (n_{H1})^2] + \rho_L [1 - (n_{L1})^2] \leq \rho_H [1 - (n_{H2})^2] + \rho_L [1 - (n_{L2})^2].
\]

The constraint is satisfied if the expected surplus does not decrease when the distribution of spending changes. The intuition is straightforward. If the expected surplus in period 1 were larger than in period 2, it would constitute a drain on liquidity, leaving insufficient liquidity after the distribution changed. The optimal tax rate would be higher starting in period 2.

In the Appendix, we also show that the constraint for period 2 is

\[
1 - (n_{H1})^2 + \beta \{ \rho_H [1 - (n_{H1})^2] + \rho_L [1 - (n_{L1})^2] \} \leq 1 - (n_{H2})^2 + \beta \{ \rho_H [1 - (n_{H2})^2] + \rho_L [1 - (n_{L2})^2] \}
\]

This puts an additional restriction on how the surplus in the high \( g \) state changes. Suppose, for example, that the expected value of the surplus remains unchanged, but that \( g_{H2} > g_{H1} \). We would then have \( n_{H2} > n_{H1} \) and a constant tax rate will result in insufficient liquidity in period 2.

The bottom line is that the liquidity constraint may bind when the distribution of government spending changes; it depends on how the distribution changes. If the constraint is

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\(^{10}\)We also assume that spending in period zero is equal to \( g_{H1} \), allowing us to avoid a complication already discussed in Example 1.
binding, the Friedman Rule is inconsistent with a constant wage tax rate.

6 Ramsey Solutions with Staggered Price Setting

In this section, we add monopolistic competition and Calvo-style staggered price setting;\footnote{Monopoly rents are taxed away using a profits tax.} we also assume that there is no indexing to steady state inflation. This creates a price dispersion that distorts household consumption decisions unless the Planner holds the aggregate price level constant, eschewing the deflation associated with the Friedman Rule and the unanticipated jumps in the aggregate price level described in the last section. As is well known, the Friedman Rule is not optimal when there are price rigidities.

In the last two sections, we saw that unanticipated jumps in the price level could be used to accommodate fiscal shocks. But if these jumps are too costly to implement, then either taxes or deficit finance must be used to accommodate fiscal shocks, and the tradeoff between liquidity provision and tax smoothing is more difficult. Access to the third debt instrument eases this tradeoff. For example, an increase in government spending can be financed by selling the illiquid bond, while still providing the level of money and bonds needed for liquidity; this allows the increase in the wage tax rate to be smoothed over time.

A substantial literature in the Barro tradition holds that short run tax distortions should be alleviated at the expense (or benefit) of a little long run consumption. In response to say a temporary increase in government spending, the government should issue new liabilities to smooth the path of the tax hikes that will ultimately be needed; long run consumption will of course have to fall a little to service the higher level of debt. Public debt and taxes would follow unit root processes as they move to efficiently absorb fiscal shocks. In this section, we will see that this fundamental insight holds in the Unconstrained Model, but not in the Constrained Model. In the Constrained Model, we will show that the Ramsey solutions are actually stationary.

We cannot solve the Ramsey problem analytically with staggered price setting. This
rigidity adds a new state variable and the math becomes intractable; we resort to numerical
methods.\textsuperscript{12} There are few parameter settings to report in our simple model. We set $\beta$ so
that the (annualized) steady state real rate of interest is 4\%. We set the Calvo parameter
so that the average price “contract” is four quarters. Finally, we let $\phi_m = \phi_b = \phi_c = 1/3$.

We will begin with a discussion of optimal taxation and inflation in the steady state.
(Since we have ruled out indexing firm prices to steady state inflation, staggered price setting
can also distort the steady state.) Then, we will show how the Ramsey Planner responds to
two shocks that have fiscal implications.

\section{Optimal Tax Rates and Inflation in the Steady State}

Table 1 reports the optimal steady state tax rates and inflation, which are the same
in the Unconstrained and Constrained models. From the last two sections, we know that
the Ramsey Planner implements the (extended) Friedman Rule when prices are flexible.
Deflation is equal to the real rate of interest. With staggered price setting, the Planner
holds the aggregate price level virtually fixed, and the seigniorage taxes are positive. Calvo
trumps Friedman in our models.

Table 1 shows that $I^c > I > 1$ in the case of staggered price setting. Why is this? If $I$ were
set equal to 1, (9) implies that the bond good - cash good margin would not be distorted, but
(10) implies that the bond good - credit good margin would be distorted. Similarly setting
$I = I^c$ would leave the bond good - credit good margin undistorted, while the bond good -
cash good margin would be distorted. Optimal policy spreads the consumption distortions
by setting $I^c > I > 1$.

\section{Response to a Temporary Increase in Government Purchases}

Figure 1 shows the Ramsey Planner’s response to an increase in government spending
that lasts just one period. We begin with the Unconstrained Model, and the solid lines.

\textsuperscript{12}More precisely, we use the \textit{Get Ramsey} program developed by Levin and Lopez-Salido (2004).
Jumps in the price level (not shown) are very costly, and the Planner makes very little use of them. Instead, the Planner issues new liabilities to avoid a big wage tax hike. The Planner can do this without distorting consumption decisions since the government has access to the illiquid debt instrument $b_g^c$. More precisely, the Planner can create any level of gross liabilities, $m + b$, by buying or selling $b_g^c$; then, ordinary open market operations can create the right mix between money and bond balances. After a few periods, the wage tax levels out at a permanently higher rate, collecting the new revenue that is needed to service the increase in net liabilities. The increase in government spending initially crowds out consumption, and work initially increases in response to the higher long run tax burden. But after a few periods, both consumption and work assume permanently lower paths, reflecting the permanently higher wage tax rate. The fundamental insight attributed to Barro is preserved; net liabilities, consumption, and the wage tax rate exhibit unit root behavior.\footnote{Of more minor note, the wage tax actually falls a little bit in the first period. The reason for this is that both seigniorage tax rates rise a little in the first period. The Planner lowers the wage tax to smooth the distortions that this creates in the labor-leisure margins, (11).}

In the Constrained Model, the government lacks an illiquid debt instrument, and the optimal response to a temporary increase in government spending is quite different, as illustrated by the dashed lines. The Planner does not issue more liabilities to avoid a big wage tax hike. Unlike in the Unconstrained Model, the Planner cannot use the third debt instrument to separate liquidity provision from deficit finance. Using deficit finance to smooth the wage tax would raise $m + b$. And since the cash and collateral constraints are binding, this would raise cash and bond good consumption; this distortion of consumption decisions would be too costly. Optimal debt management in the Constrained Model must trade off the benefits of wage tax smoothing with the costs of distorting consumption. An immediate tax hike is optimal, and fluctuations in consumption and work are much larger than in the Unconstrained Model.

Moreover, the Planner returns to the initial steady state solution. Why doesn’t the Planner do just a little of what was optimal in the Unconstrained Model? Suppose the
Planner did, in fact, permanently raise liabilities to smooth tax hikes. The long run tax burden would rise, and total consumption would have to fall in the long run. As noted above, without the third debt instrument, liquidity must rise along with liabilities (since net and gross liabilities are the same in the Constrained Model). Since the cash and collateral constraints are always binding, cash and credit good consumption would rise as well. But, as we noted, total consumption must be permanently lower, and this would mean that credit good consumption would fall a lot; this is not optimal. Liabilities and the wage tax rate return to their original steady state values, and the unit root behavior is lost.

6.3 Response to a Temporary Decrease in Productivity

Figure 2 shows the Ramsey Planner’s response to a decrease in productivity that lasts just one period. The patterns in Figure 2 are remarkably similar to those in Figure 1. There is of course a reason for this: the decrease in productivity has similar implications for both the feasibility condition and the fiscal budget as does the increase in government spending. An increase in government spending decreases (for a given work effort) the amount of goods available for consumption; similarly, a decrease in productivity means (for a given work effort) there is less output available for consumption. An increase in government spending ultimately requires an increase in the wage tax rate; similarly, a decrease in productivity lowers the real wage (which is equal to $z$ in our models), and thus wage tax revenues; ultimately this requires an increase in the wage tax rate. So, the explanations for the patterns in Figure 2 are basically the same as those for the patterns in Figure 1.

7 Conclusion

In this paper, we have extended a standard model – with cash goods, credit goods, and a distortionary wage tax – to allow for the fact that government bonds provide liquidity services. And we have shown that if the government has access to a third debt instrument,
an instrument that does not provide liquidity services, then standard results about optimal
debt management and tax smoothing survive largely intact. If the government does not
have access to such an instrument, the Ramsey Planner’s choices can be very different, and
optimal liquidity provision can conflict with tax smoothing even with flexible prices. We
have already listed our specific findings in the introduction; they need not be repeated here.

Our work suggests there is an optimal level of public debt, and a natural question to ask
is whether there is already a sufficient amount of debt available in most developed countries.
Our model, with its artifice of a bond good requiring collateral, is obviously not ready to
be taken to the data. But, the ratio of government debt to total consumption is greater
than one in most advanced economies. However, the need for liquidity provided by financial
assets extends far beyond collateral for a subset of consumption goods. A useful extension
of the work presented here would develop a framework that could be taken to the data.

How should one think about the illiquid debt instrument in our model? We model it as
government holdings of private sector bonds, which by our assumptions cannot be used as
collateral, and therefore do not provide liquidity services. One possibility is to consider it an
analytical device that highlights why liquidity provision and tax smoothing may come into
conflict. It would not seem that governments currently utilize an illiquid debt instrument,
and they certainly do not use private sector debt in the way our Ramsey Planner would.
And if they did actively trade private sector debt, that debt might possibly take on some
of the liquidity characteristics of government debt. Governments, of course, have non-liquid
assets such as parks, oil rights, grazing rights, spectrum rights, and state owned enterprises.
However these assets would be hard to use to finance short run deficits without creating
disruptions in the real economy.

Another possibility is to think of short-term government debt as liquid and take long
term government bonds as the illiquid debt instrument that is needed here, as in Greenwood
et al. (2014). But some long term debt, especially the United States Treasury’s ten-year
note, is actively used in financial markets and provides significant liquidity services. A third
possibility is to consider recent central bank purchases of private sector debt – such as asset backed securities – as an example of government holdings of illiquid claims on the private sector. Central banks appear, however, to be concerned about the potential for disruptions in private markets should these assets be sold, suggesting that these assets may not be bought and sold without any direct impact on the economy.

More generally yet, the substitutability between money and bonds in the provision of liquidity could be explored further. This could be done by say increasing the substitutability of the cash and bond goods in utility. As the cash and bond goods became perfect substitutes, money would presumably be crowded out. Or alternatively, holding the elasticity of substitution constant, one could explore the implications of the central bank’s paying interest on reserves. This could lead to a discussion of the optimal approach to a winding down of the unconventional policies pursued by central banks in recent years.

References


Canzoneri, M., Cumby, R., Diba, B., 2010. The interaction between monetary and fiscal


Table 1: Steady State Tax Rates and Inflation (annual rates)

<table>
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<tr>
<th>Flexible Prices</th>
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<th>$I^c - I$</th>
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<th>inflation</th>
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<td>-0.04</td>
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</table>
Figure 1: Ramsey Response to a Positive Government Spending Shock

Unconstrained Model: solid lines; Constrained Model: dashed lines
Figure 2: Ramsey Response to a Negative Productivity Shock

Unconstrained Model: solid lines; Constrained Model: dashed lines