

Euler Equations and Money Market Interest Rates:

A Challenge for Monetary Policy Models*

by

Matthew B. Canzoneri, Robert E. Cumby and Behzad T. Diba
Professors of Economics, Georgetown University

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ABSTRACT

Standard macroeconomic models equate the money market rate targeted by the central bank with the interest rate implied by a consumption Euler equation. We use U.S. data to calculate the interest rates implied by Euler equations derived from a number of specifications of household preferences. Correlations between these Euler equation rates and the Federal Funds rate are generally negative. Regression results and impulse response functions imply that the spreads between the Euler equation rates and the Federal Funds rate are systematically linked to the stance of monetary policy. Our findings pose a fundamental challenge for models that equate the two.

Corresponding author: Matthew Canzoneri
Economics Department
Georgetown University
Washington, DC 20057
email: canzonem@georgetown.edu
phone: 202 687-5911; fax: 202 687-6102

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I. Introduction

The consumption Euler equation of a representative household is a fundamental building block of many macroeconomic models, including the New Neoclassical Synthesis (NNS) models that are now a standard framework for the analysis of monetary policy.¹ NNS models typically equate the money market rate targeted by the central bank with the interest rate in the Euler equation; thus the Euler equation provides a direct link between monetary policy and consumption demand. In this paper, we use U.S. data to calculate the interest rate implied by the Euler equation, and we compare this Euler equation rate with a money market rate. We find the behavior of the money market rate differs significantly from the implied Euler equation rate. This poses a fundamental challenge for models that equate the two rates.

The fact that the two interest rate series do not coincide – and that the spread between the Euler equation rate and the money market rate is generally positive – comes as no surprise; these anomalies have been well documented in the literature on the “equity premium puzzle” and the “risk free rate puzzle”.² And the failure of consumption Euler equation models should come as no surprise; there is a sizable literature that tries to fit Euler equations, and generally finds that

¹ The NNS adds monopolistic competition and nominal inertia to the Real Business Cycle paradigm. Woodford (2003) provides a masterful introduction to NNS models. Christiano, Eichenbaum and Evans (2005) provide an estimated NNS model that explains the effects of a monetary policy shock well. Influential monetary policy analyses include King and Wolman (1999) and Erceg, Henderson and Levin (2000). Many central banks are now developing large scale NNS models.

² Giovannini and Labadie (1991) showed that the spread between Euler equation rates and money market rates was about as large as the equity premium. Weil (1989) illustrated what he called the “risk free rate puzzle”: combining consumption growth with the Euler equation of a representative consumer with standard, additively separable utility implies a real interest rate that is much greater than observed money market rates. In addition, Rose (1988) and others show that standard consumption Euler equations cannot explain the persistence of real short term interest rates.

the data on returns and aggregate consumption are not consistent with the model.³

If the spread between the two rates were simply a constant, or a constant plus a little statistical noise, then the problem might not be thought to be very serious. The purpose of this paper is to document something more fundamental – and more problematic – in the relationship between the Euler equation rate and observed money market rates. In Section II, we compute the implied Euler equation rates for a number of specifications of preferences and find that they are strongly negatively correlated with money market rates.⁴ This suggests that something, or some things, are systematically moving the two rates in opposite directions. One possible explanation is apparent in the figures we present in Section II. During the Volcker tightening in the early 1980's, the Euler equation rates fell while the money market rates rose, and during the Greenspan easing in the early 2000's, just the opposite occurred. These easily identified episodes suggest that the spread may be systematically linked to the stance of monetary policy.

In section III, we document the statistical link between the interest rate spread and the stance of monetary policy. We do this in two ways. First, we regress the spread on standard measures of the stance of monetary policy, and then we generate impulse response functions for monetary policy shocks. The regressions imply that a monetary tightening decreases the spread, and the impulse response functions imply that a monetary tightening increases the money market

³ One contribution of this paper is that it provides a potentially useful way of characterizing the extent to which the data and the models are inconsistent.

⁴ We consider standard additively separable CRRA preferences, four models of preferences with habit persistence, and recursive preferences like those proposed by Epstein and Zin (1989, 1991) and Weil (1990). The negative correlation appears to be quite robust to changes in preferences. The only exception is that some specifications of preferences with habit persistence yield interest rates that are so excessively volatile as to reduce the correlation nearly to zero.

rate and decreases the Euler equation rate.

The intuition for why the spread is systematically linked to monetary policy is clearest if the representative consumer has additively separable CRRA utility and consumption is lognormally distributed. In this case, the consumption Euler equation implies that the real interest rate is proportional to the expected growth of real consumption. The empirical literature shows that a monetary tightening has a small effect on consumption in the first quarter following the tightening. In the following few quarters, the consumption falls more rapidly so that expected consumption growth declines. A decline in expected consumption growth will reduce the real interest rate implied by the Euler equation. The empirical literature shows that money market rates respond in the opposite direction.

Changing the form of preferences can, in principle, address this problem. Adding habit persistence is an attractive alternative because doing so has proven useful in several other contexts. Consumption growth continues, however, to play a key role in the Euler equations obtained from alternative preferences. As a result this problem also plagues models with more general preferences, and the same intuition appears to apply.

Both of our results – the negative correlation between the Euler equation rate and money market rate, and the sensitivity of the spread to monetary policy – pose a major challenge for models of monetary policy that equate the Euler equation rate and the rate targeted by the central bank. In section IV, we summarize our results, we relate our work to some of the more recent literature on macroeconomic modeling, and we discuss ways in which NNS models might be modified to meet the challenge documented here.

II. Computing Real and Nominal Interest Rates

In this section we compute real and nominal interest rates implied by consumption Euler equations for a number of specifications of consumer preferences and compare them with money market rates. In each model, we assume that a representative agent chooses consumption and holdings of two riskless one-period bonds – one that pays one unit of the consumption good and one that pays one dollar. The consumer is assumed to maximize lifetime utility,

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} E_t u(C_s, Z_s)$$

subject to a sequence of budget constraints, where β is the consumer's discount factor and Z_s is the reference, or habit, level of consumption in period s .⁵ The first order conditions imply that the prices of the bonds are,

$$\frac{1}{1+r_t} = \beta \frac{E_t \left(\frac{\partial U_t}{\partial C_{t+1}} \right)}{E_t \left(\frac{\partial U_t}{\partial C_t} \right)} \quad \text{and} \quad \frac{1}{1+i_t} = \beta \frac{E_t \left(\frac{\partial U_t}{\partial C_{t+1}} \frac{P_t}{P_{t+1}} \right)}{E_t \left(\frac{\partial U_t}{\partial C_t} \right)}$$

where r_t is the real interest rate, i_t is the nominal interest rate, and P_t is the price of one unit of the consumption good. The models we consider differ in their specification of the period utility function and therefore in the implied marginal rates of substitution.

2.1 The Standard Preferences

We begin by assuming the representative agent has the standard, additively separable CRRA

⁵ The one exception is the recursive preference considered in section 2.6.

preferences (so Z_s doesn't appear in the utility function). The period utility function is,

$$u(C_t) = \frac{1}{1-\alpha} C_t^{1-\alpha},$$

where α is the coefficient of relative risk aversion. The corresponding Euler equation is,

$$(1) \quad (1+i_t)^{-1} = \beta E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\alpha} \frac{P_t}{P_{t+1}} \right]$$

Next, we follow Fuhrer and assume that the dynamics of consumption and inflation can be described by the vector autoregression (written in companion form),

$$(2) \quad Y_t = A_0 + A_1 Y_{t-1} + v_t$$

and let $c_t = \log(C_t)$ be the first element and $\pi_t = \log(P_t/P_{t-1})$ be the second element of the vector Y_t ,⁶

In addition, we assume that the error term, v_t , is iid $N(0, \Sigma)$. Under conditional lognormality the Euler equation implies that nominal interest rates are given by,

$$(3) \quad (1+i_t)^{-1} = \beta \exp \left[-\alpha (E_t c_{t+1} - c_t) - E_t \pi_{t+1} + \frac{\alpha^2}{2} V_t c_{t+1} + \frac{1}{2} V_t \pi_{t+1} + \alpha \text{cov}_t(c_{t+1}, \pi_{t+1}) \right].$$

⁶ We adopt the convention that a lower case letter denotes the log of the corresponding upper case letter except for interest rates. As in Fuhrer (2000), the other variables in the VAR are the log of the Journal of Commerce industrial materials commodity price index, the log of per capita real disposable income, the federal funds rate, and the log of per capita real nonconsumption GDP. In addition we follow Fuhrer by measuring consumption as per capita real expenditures on nondurables and services and beginning our estimation of the VAR in 1966:1. We measure inflation as the log change in the deflator for nondurables and services consumption. Unlike Fuhrer, we do not detrend consumption, income, and GDP. Instead, we include a (segmented) time trend in the VAR. In addition, we have considered a VAR without a trend and found that doing so had virtually no effect on our key results.

The expression for real interest rates is identical to (3) without the terms involving inflation.⁷

Assuming $\alpha=2$ and $\beta=0.993$ and the moments obtained from the VAR (see the appendix), we compute the implied nominal and real interest rates. The implied real rate is shown in Figure 1. The contrast between the behavior of the model-generated real rate and the observed ex-post real rate is striking.⁸ Most notably, the model-generated rate falls when the money market rate rises during the Volker disinflation and model-generated real rates are high during the late 1970s and early 1990s when market rates are low. And more recently, the model-generated rate rose in 2001 and remained high while money market rates fell and remained low. The stark difference in the behavior of the two rates can also be seen in Table 1, which presents summary statistics. The average real rate implied by the consumption Euler equation exceeds the ex-post real money market rate by nearly 480 basis points and the correlation between the two is -0.37.

As we discuss above, one reason that the model interest rate fails to mimic the behavior of money market rates is clear from (3). Following a monetary tightening consumption continues to fall for several quarters, so expected consumption growth falls. And from (3) a decline in expected consumption growth will reduce the real interest rate implied by the Euler equation. But the empirical literature shows that money market rates respond in the opposite direction. Changing preferences will change the details of the Euler equation, but we will see that the role of expected consumption growth is an enduring feature.

⁷ Log linearizing, as is common in the literature, would result in Euler equations that differ from those we obtain by assuming log normality only by a constant. In order to check the sensitivity of our results to the lognormality assumption we also take a second-order approximation to the Euler equations and find the results are nearly identical to those reported.

⁸ We have also compared the implied rate to an estimate of ex-ante real interest rates obtained by adjusting nominal rates by the one-quarter ahead forecast of inflation from the VAR, as well as nominal rates and find that all three sets of plots convey the same message.

One reason that the interest rates implied by the consumption Euler equation differ substantially from money market interest rates is that an Euler equation might not describe the consumption choices of all individuals, perhaps due to liquidity constraints. Campbell and Mankiw (1989) find that they are able to fit aggregate consumption data well by assuming one group of individuals consumes all of their disposable income while another group chooses consumption optimally over time (without liquidity constraints). The consumption of the first group represents roughly half of aggregate disposable income. In order to determine if this explains the results in Figure 1 and Table 1, we assume the consumption of the optimizing individuals is $c_t^* = \log(C_t - 0.5 Yd_t)$ and use c_t^* in our VAR and in the computation of the Euler equation interest rates. We find that doing so reduces the correlation coefficient between the real Euler equation rate and the ex-post real money market rate from -0.37 to -0.17, but the behavior of the two rates still differs substantially (as the negative correlation coefficient suggests).

2.2 Fuhrer's Model

Fuhrer (2000) assumes that the representative consumer's period utility function is,

$$u(C_t, Z_t) = \frac{1}{1-\alpha} \left(\frac{C_t}{Z_t^\gamma} \right)^{1-\alpha},$$

where the habit level of consumption evolves as, $Z_t = \rho Z_{t-1} + (1-\rho)C_{t-1}$ and $0 \leq \gamma \leq 1$ is a parameter indexing the importance of habit. When he estimates his model, Fuhrer finds that the estimate of ρ is close to zero and insignificant, so that his period utility function is,

$$u(C_t, Z_t) = \frac{1}{1-\alpha} \left(\frac{C_t}{C_{t-1}^\gamma} \right)^{1-\alpha} = \frac{1}{1-\alpha} \left(\frac{C_t}{C_{t-1}} C_{t-1}^{1-\gamma} \right)^{1-\alpha}.$$

Unlike in standard models, utility is not separable over time because the current period's choice of consumption affects next period's utility. Alternatively, current utility depends on both the current level of consumption and on the growth of consumption from last period. As a result, consumers will want to smooth both consumption and its growth rate.

Nominal interest rates in Fuhrer's model are then given by,

$$(4) \quad \beta(1+i_t) = \frac{E_t \left[C_{t-1}^{-\gamma(1-\alpha)} C_t^{-\alpha} - \beta\gamma C_{t+1}^{1-\alpha} C_t^{-\gamma(1-\alpha)-1} \right]}{E_t \left[\left(C_t^{-\gamma(1-\alpha)} C_{t+1}^{-\alpha} - \beta\gamma C_{t+2}^{1-\alpha} C_{t+1}^{-\gamma(1-\alpha)-1} \right) \left(P_t / P_{t+1} \right) \right]},$$

and real interest rates are defined similarly. The assumption that consumption and inflation are conditionally log normal implies that equation (4) becomes,

$$\beta(1+i_t)^{-1} = \frac{\exp(a_t) - \beta\gamma \exp(b_t)}{\exp(d_t) - \beta\gamma \exp(e_t)},$$

where,

$$a_t = \gamma(\alpha - 1)c_{t-1} - \alpha c_t$$

$$b_t = (\gamma(\alpha - 1) - 1)c_t + (1 - \alpha)E_t c_{t+1} + \frac{(1 - \alpha)^2}{2} V_t c_{t+1}$$

$$d_t = \gamma(\alpha - 1)c_t - \alpha E_t c_{t+1} - E_t \pi_{t+1} + \frac{\alpha^2}{2} V_t c_{t+1} + \frac{1}{2} V_t \pi_{t+1} + \alpha \text{cov}_t(c_{t+1}, \pi_{t+1})$$

$$e_t = (\gamma(\alpha - 1) - 1)E_t c_{t+1} + (1 - \alpha)E_t c_{t+2} - E_t \pi_{t+1} + \frac{(\gamma(\alpha - 1) - 1)^2}{2} V_t c_{t+1} + \frac{(1 - \alpha)^2}{2} V_t c_{t+2} + \frac{1}{2} V_t \pi_{t+1} \\ + (1 - \alpha)(\gamma(\alpha - 1) - 1) \text{cov}_t(c_{t+1}, c_{t+2}) - (1 - \alpha)(\text{cov}_t(\pi_{t+1}, c_{t+2}) + \text{cov}_t(\pi_{t+1}, c_{t+1}))$$

The corresponding expression for real interest rates excludes the terms involving inflation. Again, using the conditional moments from the VAR, and the parameter values reported by Fuhrer (2000), we can compute the time series of real and nominal interest rates implied by Fuhrer's model.

As can be seen from Figure 2 and Table 1, the time series of real interest rates implied by Fuhrer's model bear little resemblance to observed (ex-post) real money market rates. The average real rate computed from the model is about twice that computed from the data and the standard deviation of the real rate computed from the model is about 13 times that computed from the data. Even more striking is the range of variation of the real rates. The model implies that real interest rates vary from a minimum of less than -75 percent to a maximum of more than 95 percent per annum.⁹

The reason adding habit persistence raises interest rate variability is that it strengthens the desire for a smooth path of consumption. Greater interest rate movements are therefore needed to induce consumers to overcome their desire for smooth consumption and willingly accept a given volatility of consumption. In a full, general equilibrium model, Jermann (1998) shows that introducing an elastic supply of capital can reduce interest rate variability, but would do so at the expense of greatly reducing consumption volatility. Restoring realistic consumption variability by introducing costs of adjusting the capital stock re-introduces excessively volatile interest rates.

Another way of stating this feature of habit models is that interest rates are often quite

⁹ We experimented with a smaller VAR to see if doing so would reduce the variability in our estimates of expected one-quarter-ahead consumption growth and therefore reduce the variability of the implied interest rates. We found that doing so had little effect.

sensitive to changes in the path of consumption. As a result, there is often a trade-off between realistic interest rate behavior and realistic consumption behavior.

Fuhrer’s model of habit is hardly alone – the same problem is shared by nearly all habit models. We focus on Fuhrer’s model because it has been successful in other regards. Despite these other successes, our calculations suggest that these models are missing something fundamental about the behavior of money market interest rates. We next turn to four other preference specifications.

2.3 A “Difference” Model of Internal Habit

Edge (2000), Boldrin, Christiano, and Fisher (2001), and Christiano, Eichenbaum, and Evans (2005) adopt a specification in which utility depends on the difference between current consumption and the habit level of consumption (rather than the ratio),

$$u(C_t, Z_t) = \log(C_t - bC_{t-1}).$$

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are then given by,

$$\beta(1+i_t) = \frac{E_t \left[\frac{1}{C_t - bC_{t-1}} - \frac{b}{C_{t+1} - bC_t} \right]}{E_t \left[\left(\frac{1}{C_{t+1} - bC_t} - \frac{b}{C_{t+2} - bC_{t+1}} \right) \frac{P_t}{P_{t+1}} \right]}$$

(5)

Because this specification uses differences rather than ratios, we cannot use conditional lognormality and the VAR estimates to compute the expectations in (5). Instead, we compute

two sets of estimates: one in which we log-linearize (as do Edge, Boldrin, Christiano, and Fisher, and Christiano, Eichenbaum, and Evans) and a second in which we compute a second-order approximation to the Euler equation.¹⁰ As can be seen in Table 1 and Figure 3 where we report calculations using the log-linearized Euler equation and Christiano, Eichenbaum, and Evan’s parameters, the problem of excess volatility of the implied Euler equation rates also arises with this specification of habit, although the problem is not as extreme as encountered, for example, with Fuhrer’s model.¹¹ The implied Euler equation rates range from about -20 percent to 20 percent per annum and the standard deviation exceeds that in the data by a factor of two. The correlations between the model-generated rates and the money market rates are near zero for both nominal and real rates.

2.4 Abel’s Model of Catching Up with the Joneses

Like Fuhrer, Abel (1990, 1999) specifies the representative agent’s period utility function in a way that depends on the ratio of current consumption to a reference level of consumption,

$$u(C_t, Z_t) = \frac{1}{1-\alpha} \left(\frac{C_t}{Z_t} \right)^{1-\alpha} .$$

There is, however, an important difference – habit is assumed to be external, rather than internal. That is, the reference level of consumption depends on lagged aggregate consumption, rather than lagged individual consumption. In particular, Abel assumes that $Z_t = \bar{C}_{t-1}^\gamma G^{\eta t}$, where \bar{C}_{t-1} is

¹⁰ Christiano, Eichenbaum, and Evans (2005) date the expectations as of t-1. We find, as they do, that this makes no notable difference in the results.

¹¹ Those based on Edge’s and Boldrin, Christiano, and Fisher’s parameters are similar. And, as with the other models, we find the results from log linearization and a second order approximation to the Euler equation are virtually identical.

lagged aggregate consumption and G is the trend growth in reference consumption.

Assuming that habit is external greatly simplifies the representative agent's intertemporal marginal rate of substitution. In Abel's specification, nominal interest rates are then given by,¹²

$$(1+i_t)^{-1} = G^{\eta(\alpha-1)} \beta E_t \left[\left(\frac{C_t}{C_{t-1}} \right)^{\gamma(\alpha-1)} \left(\frac{C_{t+1}}{C_t} \right)^{-\alpha} \frac{P_t}{P_{t+1}} \right] .$$

Assuming conditional log normality,

$$(6) (1+i_t)^{-1} = G^{\eta(\alpha-1)} \beta \exp \left[-\gamma(\alpha-1)c_{t-1} + (\alpha + \gamma(\alpha-1))c_t - \alpha E_t c_{t+1} - E_t \pi_{t+1} + \frac{\alpha^2}{2} V_t c_{t+1} + \frac{1}{2} V_t \pi_{t+1} + \alpha \text{cov}_t(c_{t+1}, \pi_{t+1}) \right].$$

We compute real and nominal rates using Abel's specification under two sets of assumptions. First, we assume consumption growth is iid lognormal (as in Abel (1999)) and choose parameters using Abel's algorithm, which matches the mean and variance of real money market rates to those computed from a linearized version of his model. Next, we assume consumption and inflation are conditionally lognormal and compute the conditional moments from the vector autoregression (2).

As can be seen in Figure 4A and Table 1, the iid lognormal specification does not suffer from the extreme volatility of the real interest rate found in Fuhrer's specification. The average real rate, about 6 percent at an annual rate, is too high by a factor of more than 2.5, but its standard deviation is virtually identical to that found in the data.¹³ And the wild swings in real

¹² In equilibrium individual and aggregate consumption growth rates will be equal.

¹³ The difference between the average model rate and the average market rate is somewhat surprising because the parameters are chosen to match the two. Average consumption growth and average real interest rates are higher in our sample than in Abel's, and this makes the

rates computed from Fuhrer's specification are notably missing here. The implied model rate differs significantly from the money market rate in much of the sample (particularly around the time of the recessions of 1973:4 - 1975:1 and 1990:3 - 1991:1 as well as during the late 1970s, the early-to-mid 1990s, and the period beginning in 2001), but it does appear to capture the early-1980s disinflation fairly well.

The extreme volatility reappears, however, when we allow expected consumption growth to vary over time. Although the model rates appears to exhibit less high-frequency volatility than those computed from Fuhrer's model, the standard deviation of the model's real interest rate exceeds that observed in the data by a factor of more than 25 and the rate ranges from less than -70 percent to more than 70 percent. In addition, the correlation between the implied Euler equation rates and the money market is strongly negative.¹⁴ Figure 4B shows that the lowest real interest rates implied by the model are found around the time of the Volker disinflation when money market rates were at their peak. Implied real rates are high during the low-interest-rate periods of the early 1990s and 2001 through the end of the sample. The implied real rates are also markedly negative during the 1973:4 - 1975:1 and 1990:3 - 1991:1 recessions.

In part, this reappearance of extreme volatility may be due to the use of parameters calibrated under the assumption of iid consumption growth. In order to reduce the volatility in the implied rates, we searched for parameter values that matched the average interest rate and

linearization error discussed in Abel (1999) larger.

¹⁴ The correlation coefficient is virtually identical to that we compute using power utility because γ is much smaller than α so that expected consumption growth dominates lagged consumption growth in the Euler equation.

minimized its standard deviation.¹⁵ As can be seen in Figure 4C, this alternate set of parameter values results in a considerable reduction in volatility, but the negative correlation between the money market rate and the real rate implied by the consumption Euler equation is apparent.

2.5 Campbell and Cochrane's Model of External Habit

Like Abel, Campbell and Cochrane (1999) assume that habit is external, but unlike Abel and Fuhrer, they assume that period utility depends on the difference between consumption and the or habit level (rather than the ratio of the two) and rewrite the period utility function in terms of the “surplus consumption ratio,” $S_t = (C_t - Z_t)/C_t$. In particular, they assume,

$$u(C_t, Z_t) = \frac{1}{1-\alpha} (C_t - Z_t)^{1-\alpha} = \frac{1}{1-\alpha} (C_t S_t)^{1-\alpha}.$$

Nominal interest rates are then,

$$(7) \quad (1+i_t)^{-1} = \beta E_t \left[\left(\frac{C_{t+1}}{C_t} \frac{S_{t+1}}{S_t} \right)^{-\alpha} \frac{P_t}{P_{t+1}} \right].$$

The habit level of consumption – and therefore the surplus consumption ratio – adjusts over time as aggregate consumption changes.¹⁶ Campbell and Cochrane assume the log of the surplus consumption ratio evolves as,

$$s_{t+1} = (1-\phi)\bar{s} + \phi s_t + \lambda(s_t)(E_t c_{t+1} - c_t),$$

¹⁵ We do not attempt to match the equity premium, which results in a substantial reduction in the habit parameter.

¹⁶ As in Abel (1999), in equilibrium with identical consumers, aggregate and individual consumption will be equal.

where ϕ and $\bar{s} = \log \bar{S}$ are parameters. By assuming consumption growth is iid log normal and choosing $\bar{S} = \sigma_c(\alpha/(1-\phi))^{1/\alpha}$, $1+\lambda(s_t) = (1/\bar{S})(1-2(s_t-\bar{s}))^{1/\alpha}$ Campbell and Cochrane obtain a constant real rate.

$$(1+r_t)^{-1} = \beta \exp \left[-\alpha(E_t c_{t+1} - c_t) - \alpha(\phi - 1)(s_t - \bar{s}) + \frac{\alpha^2 \sigma_c^2}{2} (1 + \lambda(s_t))^2 \right] = \beta \exp \left[-\alpha(E_t c_{t+1} - c_t) + \frac{\alpha}{2}(1 - \phi) \right].$$

As the consumption nears the habit level, the log of the surplus consumption ratio approaches negative infinity. By assuming that the effect of consumption uncertainty (σ_c^2) rises as the log surplus consumption ratio falls, Campbell and Cochrane are able to engineer risk premia – and, therefore, a desire for precautionary saving – that rise sharply as consumption approach its habit level. This precautionary savings motive moves in a way that exactly offsets effects of desired intertemporal substitution on real interest rates, leaving real rates constant if consumption growth is iid.

As with the other models, we assume that consumption and inflation are conditionally log normal and compute real and nominal interest rates.¹⁷ The results are summarized in Table 1 and in Figure 5. The average real and nominal rates implied by the Campbell-Cochrane specification are quite close to the average rates in the data and, as expected, the problem of excessive volatility is solved.¹⁸ In fact, the volatility of both rates is below that observed in the data. Although the Campbell-Cochrane specification succeeds in eliminating the problem of excessive volatility in interest rates, money market rates and the implied consumption Euler equation rates

¹⁷ We use the parameter values chosen by Campbell and Cochrane in our calculations.

¹⁸ The average rates are also quite close to those computed under the assumption that consumption growth is iid and lognormal.

are negatively correlated.¹⁹ This negative correlation is readily seen in Figure 5. The model rates and market rates diverge sharply during the Volker disinflation and differ substantially in a number of other periods. For example, the model rates rise in the late 1970s when market rates decline, decline in the early 1980s and, again in the late 1980s, when market rates rise.

2.6 Tallarini's Recursive Preferences

Next we consider an alternative specification of preferences that, like Campbell and Cochrane's, does not suffer from the problem of excessive volatility. These preferences, suggested by Tallarini (2000), follow Epstein and Zin (1989, 1991) and Weil (1990) but set the intertemporal elasticity of substitution equal to one. Tallarini finds that including them in a real business cycle model allows the model to resolve the equity premium and riskless rate puzzles without adversely affecting the model's ability to match the volatilities and correlations of aggregate quantities. Preferences are represented by,

$$u_t = \log(c_t) + \beta \frac{1}{(1-\beta)(1-\chi)} \log\left(E_t \left[\exp\left\{ (1-\beta)(1-\chi)u_{t+1} \right\} \right]\right)$$

where β is the discount factor and χ is the coefficient of relative risk aversion. The Euler equation is then,

$$(1+r_t)^{-1} = E_t \left[\beta \frac{c_t}{c_{t+1}} \frac{\exp\left((1-\beta)(1-\chi)u_{t+1}\right)}{E_t \left[\exp\left((1-\beta)(1-\chi)u_{t+1}\right) \right]} \right]$$

¹⁹ The correlation coefficient is virtually identical to that obtained with power utility because both Euler equations depend on expected consumption growth plus a constant.

We set $\beta = 0.993$ and $\chi = 5$ and compute the moments of the real Euler equation rate. The mean real Euler equation rate matches the data fairly well and, as with the Campbell-Cochrane model, there is no problem of excessive volatility. The volatilities of the Euler equation rates are below those observed in the data.

These preferences fail, however, to resolve the correlation problem that we have found with other preferences. When we compute a log-linear approximation to the Euler equation we find that the correlation between the implied real Euler equation rate and the real money market rate is -0.429 .²⁰

III. The Response of Model and Market Interest Rates to Monetary Policy Shocks

The results in Section II suggest that some thing, or some things, are moving the two rates in opposite directions. The figures presented in Section II also suggest one possible explanation: the spread between money market interest rates and implied Euler equation rates appear to respond to monetary policy. During periods of monetary tightness, such as the Volker disinflation, the money market rate rises and the Euler equation rate falls. And during periods of monetary easing, the opposite occurs. Although the figures are suggestive, a number of shocks in the VAR influence consumption and inflation. Hence a variety of factors may be driving the negative correlations we report.

In this section we explore the links between the spread and monetary policy in two ways. First we consider regressions of the spread on two indicators of monetary policy. We then use an identified VAR to examine the effect of a shock to monetary policy on consumption and inflation, and on the Euler equation rate implied by the response of consumption and inflation.

²⁰ The correlation computed using a second-order approximation is -0.431 .

We begin with regressions of the spread (defined as the model-generated interest rate less the money-market interest rate) on four lags of the spread and (individually) two measures of monetary policy suggested by Christiano, Eichenbaum, and Evans (1999). The first is the federal funds rate and the second is the ratio of nonborrowed reserves plus extended credit to total reserves (which we refer to as the S-ratio, after Strongin (1995)).

The regression results are reported in Table 2. The results for real and nominal rates are virtually identical – both show that monetary expansions are associated with a wider spread (model rate - market rate). The estimated coefficients for the federal funds rate are negative for all of the preferences so the measured spread widens when monetary policy eases. The coefficients are highly significant for power utility and for three of the five habit models. The coefficient for Fuhrer’s model is significant at only the 14 percent level while the coefficient for Edge’s model is significant at the 8 percent level. Given the extreme volatility of rates computed from Fuhrer’s model, it is not surprising that the coefficient is less precisely measured.

The results from the regressions using the S-ratio are roughly similar. Each of the estimated coefficients is positive, again indicating that a monetary expansion (an increase in the S-ratio) widens the measured spread. These results all suggest that the habit models, like the standard CRRA models, are missing something systematic about the way that monetary policy influences real and nominal interest rates.

Next we use the unrestricted VAR to compute the response of the implied Euler equation rate to a monetary policy shock for two sets of preferences: standard additively-separable preferences with constant relative risk aversion and the “difference” specification of preference

with habit persistence.²¹ Figure 6 contains the responses of four series to a Federal funds rate shock: the response of consumption (from the unrestricted VAR), the implied response of the real Federal funds rate (computed from the responses of the nominal Federal funds rate and inflation from the unrestricted VAR), and the implied responses of the real Euler equation rates for the two specifications of preferences (both computed from the response of consumption from the unrestricted VAR). The 95 percent confidence intervals for the impulse responses in the figure are computed from 1,000 replications using Kilian's (1998) bias-corrected bootstrap procedure. The hump-shaped response of consumption is apparent in the figure.

Neither specification of preferences generates an impulse response function for the Euler equation rate that resembles that of the real money market rate. In fact, the money market and Euler equation rates appear to move in opposite directions following a monetary policy shock – a positive Federal funds rate shock leads to a statistically significant fall in both Euler equation rates. The impulse response for the Euler equation rates implied by additively separable preferences with CRRA is nearly a mirror image of that for the money market rate. Following a positive shock to the Federal funds rate, the real money market rate declines slowly. In contrast, the Euler equation rate falls on impact and then rise slowly. The impulse response function implied by the difference specification of preferences with habit persistence also differs markedly from that of the money market rate. The real Euler equation rate falls on impact but the decline lasts only one period.

IV. Summary and Discussion of the Findings

²¹ We choose the first because it is widely used and the second because Christiano, Eichenbaum, and Evans (2005) report that their model, which includes these preferences, “does well at accounting for the dynamic response of the U.S. economy to a monetary policy shock.”

Interest rates implied by combining the dynamics of consumption and inflation observed in U.S. data with Euler equations derived from several specifications for preferences exhibit behavior that differs strikingly from that of money market interest rates. We first showed that the rates implied by consumption Euler equations and the money market rate are not highly correlated. Instead, the correlation between the two rates is strongly negative, except for preferences that imply the Euler equation rate is extremely volatile, which virtually eliminates any correlation at all. This result raises a problem for standard macroeconomic models, which equate the Euler equation rate with the money market rate. Next we showed that the difference between the implied consumption Euler equation rates and the Federal Funds rate is systematically related to monetary policy. This second result raises a problem that is especially severe for models, such as NNS models, that examine the effects of monetary policy.

A large empirical literature shows that monetary policy has a liquidity effect – that is, an unexpected monetary tightening raises nominal and real money market rates. The same literature finds that a monetary tightening reduces consumption and its rates of growth for several quarters. In this paper we showed that it is difficult to reconcile these two facts with models that equate Euler equation rates with money market rates. The problem arises because a decline in expected consumption growth appears to be associated with a decline in real interest rates in all of the Euler equations we considered; adding habit formation to consumer preferences does not seem to change this basic result. Neither do the recursive preferences used by Tallarini (2000) and Epstein and Zin (1989, 1991). It is of course possible that some other preferences (with or without habit) could resolve the puzzle, but doing so would require that the impact of expected consumption growth be reversed.

Adding habit formation to consumer preferences has yielded a significant payoff in

recent macroeconomic modeling. Fuhrer (2000) and Christiano, Eichenbaum, and Evans (2005) find that habit persistence is instrumental in allowing their models to generate macroeconomic responses to monetary policy shocks that are consistent with the responses found in unrestricted VARs. Boldrin, Christiano, and Fisher (2002) find that habit persistence allows their two sector model to generate a mean riskless rate and equity premium consistent with those observed in the data; it also improves their model's ability to reproduce key aspects of business cycles. And Tallarini (2000) finds that including modified Epstein-Zin preferences allows his real business cycle model to resolve the equity premium and riskless rate puzzles without adversely affecting the moments of the model's aggregate quantities.

In light of this success, our results may come as a bit of a surprise. Of course, we are looking at a correlation that has not been considered previously – the correlation between a model's Euler equation rate and an observed money market rate. But still, it is worth thinking about how our results might, or might not, be consistent with the recent literature on macroeconomic modeling with habit formation.

First, there is no necessary conflict between Christiano, Eichenbaum, and Evans' (2005) results and our finding of excessive volatility in the Euler equation rate implied by their specification of preferences. Indeed, the volatility in that rate is no surprise: Boldrin, Christiano, and Fisher (2001) note that the very same preferences yield excessive volatility, but they question whether this fact will ultimately prove to be a problem for models with habit, citing the work of Abel (1999) and Campbell and Cochrane (1999). Our results suggest that the problem is fundamental: alternative specifications of preferences can eliminate the excessive volatility, but they yield an Euler equation rate that is strongly negatively correlated with the money market rate.

Second, our finding that the spread between the Euler equation rate and the money market rate responds systematically to the stance of monetary policy might at first appear to be inconsistent with the success reported by Christiano, Eichenbaum, and Evans (2005). However, we think there is an explanation for the difference in our results, and it is rather subtle. The difference lies in the fact that our calculations use the impulse response for consumption from the unrestricted VAR. In contrast, Christiano, Eichenbaum, and Evans estimate the parameters of their model so as to match as closely as possible all of the impulse responses to a monetary policy shock. In doing so they trade off deviations in the impulse responses of both consumption and interest rates from those found in the unrestricted VAR. The model's interest rate response can be brought closer into alignment by allowing the consumption response to differ from that in the unrestricted VAR. And given the sensitivity of interest rates to the path of consumption that is characteristic of models with habit, relatively small differences in the path of consumption can have large effects on the Euler equation rate.

Consider, for example, the impulse response function of the real Euler equation rate implied by the preferences used by Edge (2004) and Christiano, Eichenbaum, and Evans (2005). As we note above, the impact effect of a positive federal funds rate shock on the Euler equation rate is significantly negative. In fact, not a single one of the 1,000 bootstrap replications produce a positive initial response. Nonetheless, we can produce a positive initial Euler equation response – indeed we could produce one that is identical to the change in the real Federal funds rate – by judiciously choosing consumption responses well within our estimated confidence bands.

In this paper, we have tried to resolve the problem posed by the failure of implied Euler equation rates to match the behavior of money market rates by changing consumer preferences.

As an alternative, Daniel and Marshall (1997, 1998) note several possible frictions may account for the failure of consumption-based models to produce either an equity premium or a riskless money market rate that correspond to those observed in the data. They argue that if market frictions of some kind are behind these problems then the failures ought to be more prevalent in quarterly data than in annual or biennial data. They then show that models with habit persistence do a considerably better job of matching the mean and variance of both the equity premium and the riskless rate at one-year and two-year horizons. We have also considered one-year and two-year horizons and found that the negative correlation problem is reduced, but not completely eliminated. If market frictions are important, as these results suggest, then modeling these frictions is essential for monetary policy models, which necessarily focus on relatively high (quarterly) frequency fluctuations in interest rates and macroeconomic aggregates.

Limited participation models and models attributing liquidity services to money market assets are two alternatives within the representative agent paradigm.²² Both model a wedge between the CCAPM rate and the money market rate. Lucas (1990), Fuerst (1992), and Christiano and Eichenbaum (1992, 1995, 1997) assume that households do not adjust their money holdings immediately following a monetary policy shock. Instead, the impact of a monetary shock falls on financial intermediaries, which, in turn, adjust their lending to firms. As a result, money market interest rates are no longer given by a consumption Euler equation.

Bensal and Coleman (1996), Canzoneri and Diba (2005), and Canzoneri, Cumby, Diba, and López-Salido (2006) introduce a spread by allowing bonds to provide transactions services. In Canzoneri and Diba's model, an expansionary open market operation increases the ratio of

²²A third alternative is heterogeneous agent models with liquidity constraints (Huggett and Ospina (2001)).

money to bonds; this in turn lowers the money market interest rate by changing the marginal transactions services of money and bonds. In practice, it remains to be seen if this prediction is empirically significant.

Appendix

The conditional moments that we use to compute real and nominal interest rates are obtained from the vector autoregression,

$$Y_t = A_0 + A_1 Y_{t-1} + v_t,$$

where $c_t = \log(C_t)$ and $\pi_t = \log(P_{t+1}/P_t)$ are the first and second elements of the vector Y_t and the error term, v_t , is iid $N(0, \Sigma)$.

Conditional Expectations. The expectation of one-period-ahead and two-periods-ahead consumption and inflation are just the first and second elements of the vectors,

$$E_t Y_{t+1} = A_0 + A_1 Y_t,$$

$$E_t Y_{t+2} = A_0(I + A_1) + A_1^2 Y_t.$$

Conditional Variances and Covariances. The conditional second moments are constant and given by the 1,1 and 2,2, and 1,2 elements of,

$$V_t(Y_{t+1}) = \Sigma,$$

$$V_t(Y_{t+2}) = A_1 \Sigma A_1' + \Sigma,$$

$$C_t(Y_{t+1}, Y_{t+2}) = \Sigma A_1'.$$

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Table 1
Summary Statistics for Real and Nominal Interest Rates
(percent per annum)

		Rates Computed from Models					
Real Rates	Data	CRRA	Fuhrer	Abel	Campbell - Cochrane	Christiano- Eichengreen -Evans	Abel (iid)
Mean	2.32	7.08	5.66	8.34	2.20	2.10	6.14
Std Deviation	2.39	1.66	31.25	26.55	1.64	7.39	2.32
Minimum	-2.54	1.64	-75.67	-70.99	-3.18	-18.49	0.59
Maximum	11.53	10.63	95.15	70.32	5.70	21.73	15.43
Corr(data, model)		-0.37	-0.07	-0.36	-0.37	-0.09	0.17
Nominal Rates	Data	CRRA	Fuhrer	Abel	Campbell - Cochrane	Christiano- Eichengreen -Evans	
Mean	6.76	11.56	10.11	12.80	6.66	6.48	
Std Deviation	3.27	1.98	31.54	25.88	1.95	7.47	
Minimum	1.00	7.46	-68.64	-63.19	2.58	-9.91	
Maximum	17.78	16.28	105.35	73.10	11.33	31.10	
Corr(data, model)		0.20	-0.10	-0.61	0.19	0.01	

Table 2
Response of Interest Rate Spreads to Monetary Policy
(standard errors in parentheses)

Real Rates						
Monetary Policy Indicator	CRRRA	Fuhrer	Abel	Campbell - Cochrane	Edge	Christiano-Eichengreen-Evans
Federal Funds Rate	-0.482 (0.064)	-1.215 (0.825)	-1.714 (0.491)	-0.482 (0.064)	-0.749 (0.427)	-0.586 (0.214)
S-Ratio	0.062 (0.015)	0.263 (0.214)	0.346 (0.103)	0.062 (0.015)	0.133 (0.109)	0.079 (0.052)
Nominal Rates						
Monetary Policy Indicator	CRRRA	Fuhrer	Abel	Campbell - Cochrane	Edge	Christiano-Eichengreen-Evans
Federal Funds Rate	-0.357 (0.047)	-1.035 (0.826)	-1.463 (0.461)	-0.361 (0.047)	-0.652 (0.425)	-0.533 (0.214)
S-Ratio	0.048 (0.010)	0.258 (0.215)	0.322 (0.098)	0.048 (0.010)	0.132 (0.109)	0.079 (0.051)

Notes: In each regression, the spread (defined as the interest rate computed from each model's Euler equation less the federal funds rate) is regressed on four lags of the spread and one of the two indicators of monetary policy. Only the coefficients and corresponding standard errors for the monetary policy variables are reported.

Figure 1: Real Interest Rates
Ex Post and CRRA Euler Equation

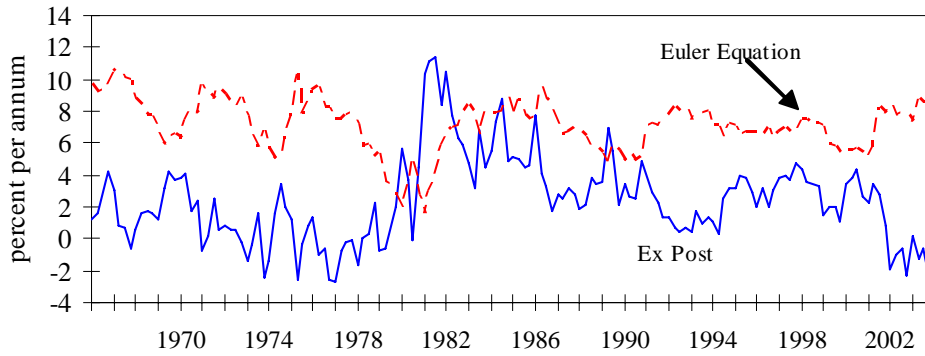


Figure 2: Real Interest Rates
Ex Post and Fuhrer Euler Equation

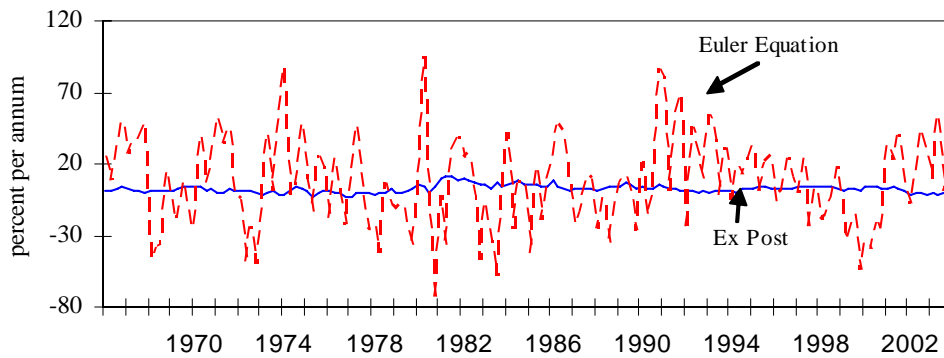


Figure 3: Real Interest Rates
Ex Post and C-E-E Euler Equation

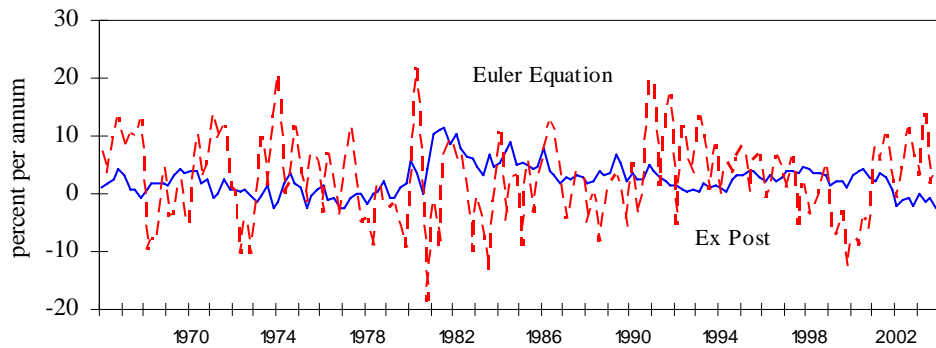


Figure 4A: Real Interest Rates
Ex Post and Abel Model (iid)

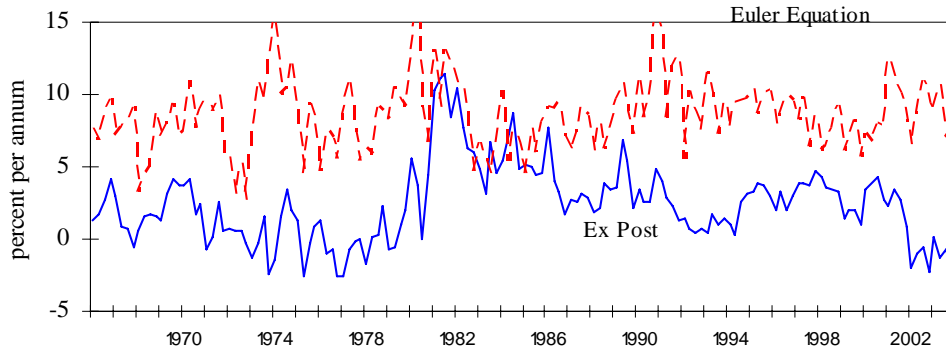


Figure 4B: Real Interest Rates
Ex Post and Abel Model

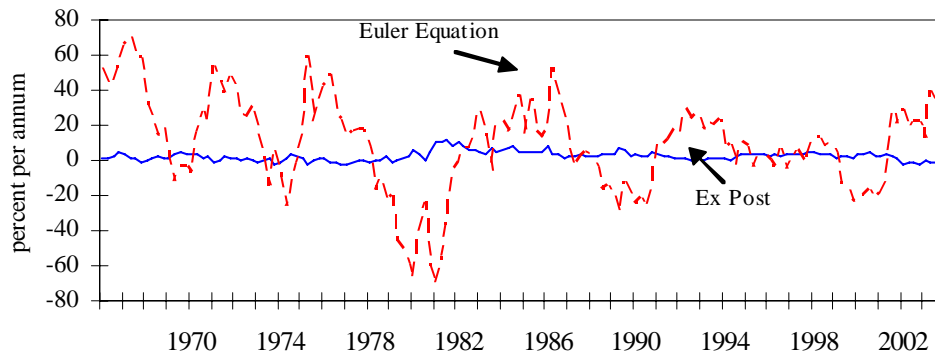


Figure 4C: Real Interest Rates
Ex Post and Abel Model (alternate)

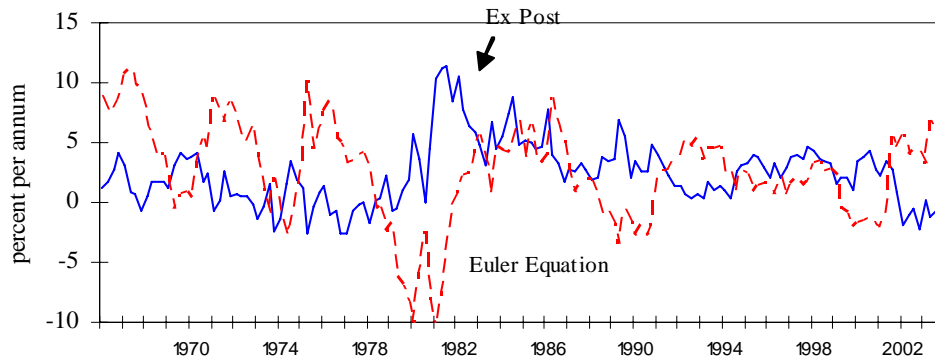


Figure 5: Real Interest Rates
Ex Post and Campbell-Cochrane Model

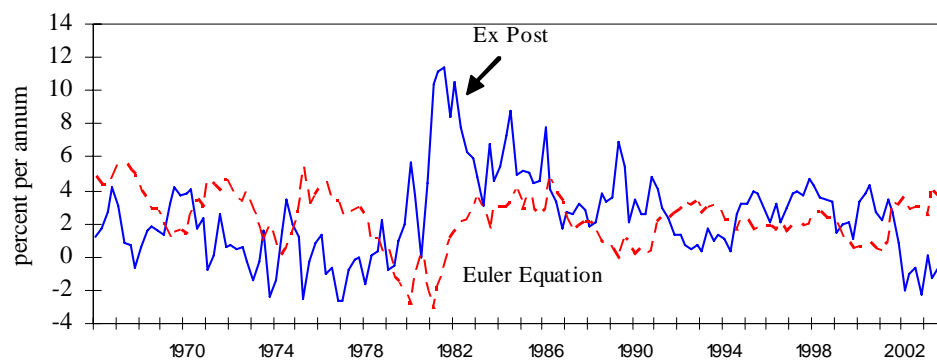


Figure 6: Impulse Response Functions for Federal Funds Rate Shock

