A search-theoretic model of the retail market for illicit drugs is developed. The model produces testable implications regarding the effect of interdiction and enforcement on: (a) the distribution of purity offered in equilibrium; and (b) the duration of the relationships between buyers and sellers. The model is consistent with evidence from the STRIDE and ADAM datasets.

1 Introduction

In this paper we build a search-theoretic model of the retail heroin market that better represents the frictions that are known to exist in illicit markets.

Existing theoretical models of the retail trade in illicit drugs are tied to traditional economic assumptions of an ideal centralized market, including assumptions of frictionless trading and perfect information (Becker Murphy & Grossman, 2006; Poret & Tejedo, 2006; Poret 2002). Of course, retail transactions of illicit drugs do not take place in an ideal centralized market and information asymmetries abound. In the retail trade, the quality of the drugs is effectively chosen by the seller (through dilution of the product) and is largely unobservable

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*We thank, without implicating, Peter Reuter who gave the impulse for the writing of this paper.

1A somewhat more sophisticated literature exists focusing on consumer demand for illicit drugs (Grossman and Chaloupka, 1998; Becker and Murphy, 1988; Schelling 1984; Stigler and Becker, 1977).
to buyers until after they consume. This moral hazard appears to be a big problem: a large fraction of the transactions, for example, are found to be pure “rip-offs” in which the buyer is sold zero-purity heroin. If this opportunistic behavior were too prevalent, buyers would presumably shy away and the market would collapse. In fact, it is remarkable that these markets exist at all. Because the drugs trade is illegal, this moral hazard is not remedied by a legally enforceable obligation, nor can sellers seek certification by a reputable intermediary. Under these circumstances, long-term relationships between buyers and sellers are key to market viability (Reuter and Caulkins 2004). These long-term relationships, which we shall document empirically, alleviate the moral hazard problem that would otherwise foreclose the possibility of trade.

Another key friction that surely affects the quality of the drugs being traded is the presence of search costs due to police enforcement. Trade in illicit substances is secretive and bi-lateral, and buyers know that searching for new trading partners is risky. Increasing these search costs induces a degree of “buyer lock-in” and so reduces the competitiveness of the market. The effects on the quality of drugs traded are complex, however, because while sellers are able to take advantage of their market power by decreasing the quality of the drugs they sell, at the same time they are able to forge a more durable relationships with their buyers.

In this paper we attempt to provide a coherent account of the effect of these two frictions—moral hazard and search costs—on the functioning of the retail market for illicit drugs. We start with a standard search model a’ la Burdett-Mortensen (1998). Searching for a new seller is costly, and consumers are sometimes forced to sample new sellers (for example, because their regular seller is unavailable). A seller always offers the same quality to all buyers. Over time, buyers who start off paired with low-quality sellers search until they find a suitably high-level quality seller, and then they form a stable match. They stick to the match until either (a) the match is exogenously broken (for example, seller goes to jail); or (b) they get to sample another seller (maybe they can’t locate their regular seller that day) who happens to sell better quality, in which case they switch. As in Burdett and Mortensen (1998), we get dispersion in the price/quality ratio in equilibrium, which is consistent with the evidence. We modify this model in a couple of ways, in order to capture some key features of the market. First, prices are presumed to be nominally fixed at amounts ($20, or $40), making variability in pure amount exchanged the key dimension of these transactions. More crucially, we assume that buyers can only determine the quality of drugs after the trade is consummated. Due to this severe moral hazard problem, in our model the market can only be sustained through long term relationships between buyers and sellers. These modification take us a long way towards accounting for some key stylized facts in the market of interest.

A point of keen interest to everyone is, of course, the suppression of the market for illicit
substances. According to our model, the retail market for illegal drugs is inherently at risk of collapse from overcutting; its viability is predicated on the presence of long-term buyer-seller relationships. This view suggests the possibility of undermining the retail drugs trade by leveraging the moral hazard problem inherent in the trade. Can we use police enforcement to undermine the viability of long-term buyer-seller relationships? While empirical evidence sufficient to definitively answer this question is not available, our model helps us work through certain effects. We find that police enforcement affects buyer-seller relationships in complex ways. Increasing the enforcement pressure on buyers, for example, may lead to more stable relationships, as buyers are led to avoid the risk entailed in searching for new trading partners. Perversely, this makes it less attractive for sellers to cheat and hence may reduce the amount of cutting and support the viability of the market. However, police enforcement that is effective in breaking up long-term relationships helps undermine the viability of the market. Adding another layer of complexity, the quality of drugs traded is also related to the degree of competition among sellers. So if enforcement increases the competitiveness of the market, for example by forcing buyers occasionally to sample a new seller, the result might well be to increase the quality of the drugs traded.

1.1 Related Literature

TO BE ADDED

2 Certain Stylized Facts About the Heroin Market

The model developed in the next section makes certain assumptions about behavior and how the market operates. Key assumptions necessary for the construction of this model are that

- there is very significant price dispersion in the market,
- buyers cannot distinguish the quality of the drugs they buy, and hence moral hazard exists in the market,
- moral hazard is tempered by the presence of long-run relationships (loyalty) between buyers and sellers as experience with the quality provided by one seller is established, and
- enforcement can interfere with these long-term relationships.

In this section we attempt to ground these key assumptions of our model into empirical evidence based on what is known about the market for heroin in the United States.
2.1 The data

Our information regarding drug markets and how buyers and sellers transact comes from two primary data sources: the System to Retrieve Information from Drug Evidence (STRIDE) database and the Arrestee Drug Abuse Monitoring (ADAM) Program.

STRIDE was created by the Drug Enforcement Agency (DEA) in the mid 1970s to record evidence about illicit drugs that are obtained from seizures, purchases and other drug acquisition activities conducted by undercover agents and informants. The data that is collected include the type of drug obtained (heroin, cocaine, marijuana, methamphetamines...), price (in the case of a purchase), city and date of acquisition, quantity as well as, very importantly, the purity level of the drug which is determined through chemical analysis in a DEA laboratory. This data set is a very useful source of information because it involves a large number of observations collected over a long period of time. Moreover, it allows for quality comparisons across transactions thanks to the information on the purity level of drugs. The main disadvantage of STRIDE is that it was not collected for research purposes but rather with law-enforcement prerogatives in mind. As a result the reported transactions are not a random sample of the underlying population. However, we believe that it is still possible to draw conclusions about some general characteristics of the drugs market so long as the limitations of the data are kept in mind.

We use information available in the 1981-2003 STRIDE which has a total of approximately 780,000 observations for a number of different drugs and acquisition methods. We concentrate our attention to the heroin market for two main reasons: purity data are available (unlike, for instance in the marijuana market where purity is not applicable); furthermore, the second data set that we use, ADAM, contains interviews where the subjects are specifically asked about their heroin consumption (see below). We have approximately 105,000 observations which are reduced to about 62,000 after dropping any method of acquisition other than purchase (e.g. seizure or “flashing money”). Several data cleaning steps are taken following Arkes et al (2004): we drop observations that contain missing values or that report zero prices or quantities; observations that are measured in quantities that are not easily translated into grams; observations acquired outside the United States borders; observations with weight that is lower than 0.1 grams (see Arkes et al (2004) for a detailed discussion of the sample restriction). This leaves 29,177 data points from 824 cities (most with very few observations) which we use in the subsequent analysis.

The ADAM data set, administered by the U.S. Department of justice, is collected quarterly from interviews with arrestees in 35 counties across the country. Individuals involved in non-drug and drug-related crimes are selected for inclusion in the study, with the goal of obtaining information about the use, importance and role of drugs and alcohol among those
committing crimes. In addition to interviewing arrestees, urine samples are requested and analyzed for validation of self-reported drug use. Since 2000, a drug market procurement module has been included as part of the quarterly survey and collects information on the arrestee’s most recent heroin purchase for all arrestees who report having used heroin in the previous 30 days. Information collected includes number of times heroin was purchased in past 30 days, number of dealers they transacted with, whether they last purchased from their regular dealer, difficulties experienced in locating a dealer or buying the drug, and the price paid for the specific quantity purchased. Although the data is collected from a non-representative sample of all heroin users, information is collected from a particularly relevant group that are likely to frequently engage in the heroin market.

2.2 Evidence of Moral Hazard Price Dispersion and Price Dispersion

To motivate our discussion of moral hazard and price dispersion, we first present a figure from the STRIDE data set that shows what was purchased in New York with $100 between 1995-97. The first graph reports the raw quantity (weight) of each transaction. The second graph shows the pure quantity in heroin, i.e. the product of raw weight with the purity level of each transaction.

We follow the criminology literature in assuming that consumers primarily care about the pure quantity that they receive and that any additional effects are second order.²

We find three features of this figure particularly striking. First, a large number of observations involve near-zero levels of purity. In these transactions the buyers parted with $100 dollars to receive a product that is at best worth a few cents. The only way to rationalize such behavior is that it is very difficult for buyers to assay the purity level of drugs at the time of purchase. This may be due to the speed at which a transaction needs to be completed or due too the inherent difficulty of telling the purity content of 0.5 grams (see Reuter and Caulkins (2004) for a discussion). Second, there is a huge amount of dispersion in pure quantities. This suggests that the market is characterized by large frictions (search frictions, as well as the informational frictions mentioned above) and that the law of one price does not hold. Below we document that this dispersion is not specific to New York and that it is not due to variation in spot prices during the chosen time period. Third, there is a discontinuity between the near-zero observations and the bulk of positive-purity observations. This suggests that the opportunistic sellers are well-aware of the fact that they are cheating their

²For instance, it could in principle be the case that a buyer prefers to purchase 1 gram that contains 20% pure heroin to 2 grams with 10% purity. We ignore such possibilities.
customers: if they intend to cheat by selling “below-par” purity they go all the way to zero which explains why there are very few observations just above zero.

Our next step is to document that these features are not specific to the place or time of purchase. The table below summarizes the importance of zero- and almost-zero purity transactions both for the full sample and for purchases worth less than 200 1983 dollars (approximately equal to $370 of 2003). Fully 9.5% of transactions worth less than $200 has purity less than 2%. We conclude that (the possibility of) cheating is a pervasive feature of this market.

Figure 1: The $100 Question
To measure price dispersion we start by noting that a heroin transaction involves three variables: price, quantity and purity. The measure of what a buyer purchases is therefore given by the product of (raw) quantity with purity. The price measure that we utilize is pure quantity per hundred dollars. We do not use the reciprocal (and perhaps more common) measure because in the case of zero-purity transactions the real price that the buyer faces is infinite which is inconvenient to deal with.

Below we report summary statistics from our sample. $100 buys on average 0.4 pure grams of heroin with a rather immense standard deviation which leads to a coefficient of variation (standard deviation over mean) of 1.46. By a way of comparison to a licit market, Treno et al (1993) calculate that the coefficient of variation for wine in California in 1990 was 0.34 without controlling for (to a large extent observable) quality.

<table>
<thead>
<tr>
<th>Sample</th>
<th>N</th>
<th>mean</th>
<th>std. dev.</th>
<th>coeff. of variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full</td>
<td>29,177</td>
<td>0.394</td>
<td>0.576</td>
<td>1.46</td>
</tr>
<tr>
<td>Restricted 1</td>
<td>20,926</td>
<td>0.370</td>
<td>0.537</td>
<td>1.45</td>
</tr>
<tr>
<td>Restricted 1 - City/Year FE</td>
<td>20,926</td>
<td>0</td>
<td>0.454</td>
<td>1.23</td>
</tr>
<tr>
<td>Restricted 2</td>
<td>12,955</td>
<td>0.405</td>
<td>0.527</td>
<td>1.30</td>
</tr>
<tr>
<td>Restricted 2 - City/Quarter FE</td>
<td>12,955</td>
<td>0</td>
<td>0.447</td>
<td>1.10</td>
</tr>
</tbody>
</table>

Of course, one might expect that any temporal or geographical difference in prices may inflate our measure of dispersion. For this reason we conduct two fixed fixed regressions on cities, a time dummy and and interactions of the two terms and compute the same dispersion measures using the residuals. In the first regression the time variable is the year and the second it is the quarter that the transaction took place. Before conducting the fixed effects regression we restrict our sample to cities that have more than 200 observations in total in the case of the year and 880 observations in the case of the quarter regressions. Furthermore we drop all transactions that belong to city-time cells with fewer than 5 observations. In the first case we are left with 20,926 data points from 23 cities and in the second with 12,955 data points from 8 cities. One should note that both our restricted samples have very similar summary statistics to the full sample. One remarkable feature of the summary statistics of

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3These restrictions are done for computational reasons.
the residuals is that the standard deviation decreased by only 17% and 15% in the year and quarter regressions respectively, even though one of the dominant stylized facts of the drugs markets in the last 25 years is the dramatic decrease in price. Our results suggest that most of the price variation occurs within a point in space and time rather than across different points. Finally in both regressions the coefficient of variation (using the mean prior to the regression which, if anything, yields an underestimate) is still more than 3 times larger than the one for wine.

### 2.3 Frequency of Consumption and Buyer Loyalty

The ADAM data provides information about how frequently buyers buy drugs. Almost 60% of arrestees admitting to heroin use in the past thirty days report buying heroin at least 28 times during the past month. In addition, over a third of these arrestees reporting use of heroin in the past 30 days report buying it multiple times in a single day. Table 1 shows the breakdown in responses to the question, “How many times did you buy heroin on the day of last purchase?” Our conclusion from this evidence is that heavy users of heroin not only consume very often but they also purchase drugs very often. This is important because it allows for the formation of long-term relationships between buyers and sellers.

<table>
<thead>
<tr>
<th>Times bought heroin on same day</th>
<th>Number</th>
<th>Frequency (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>721</td>
<td>61.6%</td>
</tr>
<tr>
<td>2</td>
<td>277</td>
<td>23.7%</td>
</tr>
<tr>
<td>3</td>
<td>121</td>
<td>10.3%</td>
</tr>
<tr>
<td>4</td>
<td>51</td>
<td>4.3%</td>
</tr>
<tr>
<td>Total</td>
<td>1,170</td>
<td>100%</td>
</tr>
</tbody>
</table>

In our model some buyers engage in long-run relationships with sellers. These long-run relationships mitigate the moral hazard implicit in the trade. The ADAM data offer some empirical measure of how much loyalty there is in the retail market. Among heroin users, the fraction who in their last transaction report dealing with a regular, occasional, or new seller are as follows.

---

4Those who reported obtaining heroin in the past 12 months.
<table>
<thead>
<tr>
<th>Source from whom you bought heroin</th>
<th>Number</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular dealer</td>
<td>787</td>
<td>67.4%</td>
</tr>
<tr>
<td>Occasional dealer</td>
<td>299</td>
<td>25.6%</td>
</tr>
<tr>
<td>New dealer</td>
<td>82</td>
<td>7.0%</td>
</tr>
<tr>
<td>Total</td>
<td>1,168</td>
<td>100.0%</td>
</tr>
</tbody>
</table>

Thus, a lot of long-run relationships are reported by these consumers, consistent with the equilibrium of our model. Indeed, we may surmise that some of the buyers who report buying from an occasional or new dealer might have tried (and failed to) locate their regular dealer, so the table may underestimate the fraction of buyers who attempt to buy from their regular dealer.

### 2.4 Turnover of Buyer-seller Matches

An additional source of friction in this market is the high turnover of the buyer-seller relationships. We distinguish between two types of turnover, permanent and temporary, both of which will feature in our model.

By permanent turnover we refer to a situation where a buyer-seller match is broken forever. This may be due to death or incarceration of either the buyer or the seller. Reuter, MacCoun and Murphy (1990) estimate that in 1988 in Washington DC the probability that a drug dealer became incarcerated was 22%. In addition, drug dealers faced a 1 in 70 annual risk of getting killed and 1 in 14 risk of serious injury.

Furthermore, temporary interruptions are also common. The ADAM data provide a glimpse of the causes that may lead a match to be interrupted. Among those who report not being able to buy heroin in the past 30 days, the breakdown of the causes is as follows.

<table>
<thead>
<tr>
<th>Why buyer reports not bought heroin in past 30 days</th>
<th>Number</th>
<th>Frequency (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No dealer available</td>
<td>92</td>
<td>33.3%</td>
</tr>
<tr>
<td>Police activity</td>
<td>77</td>
<td>28.0%</td>
</tr>
<tr>
<td>Dealer did not have any</td>
<td>52</td>
<td>18.8%</td>
</tr>
<tr>
<td>Did not have quality</td>
<td>17</td>
<td>6.2%</td>
</tr>
<tr>
<td>Other</td>
<td>38</td>
<td>13.8%</td>
</tr>
<tr>
<td>Total</td>
<td>276</td>
<td>100.0%</td>
</tr>
</tbody>
</table>

We take the view that police enforcement is an important contributor to the rate of interruption of matches. The data reported above seem to support this conclusion, especially if we believe that the reason “no dealer available” reflects to some extent police enforcement.
2.5 Summary: Model Desiderata

We view the stylized facts described above as important features that a good model of the retail market for drugs should capture. Informed by the evidence reported above, we want a model with the following characteristics.

(i) Agents trade an experience good
(ii) Consumption is frequent which allows the formation of long-term relationships
(iii) The good is addictive, which motivates our assumption of inelastic demand.
(iv) The market is illegal which means that the usual economic institutions that solve the information/experience-good problem are not present (eg, restaurant guides, consumer reports etc) leaving long-term relationships as the principal way of solving the information problem.
(v) More importantly, although the good is storable, buyers seem to consume their purchases instantaneously probably because credit constraints do not allow them to buy large quantities and/or legally they face much larger penalties if caught with higher quantities.
(vi) Illegality implies some of the market parameters (frictions) are a policy variable.

In the next section we build a model with these characteristics.

3 The Model

3.1 Basics

Time runs continuously, the horizon is infinite, and the future is discounted at rate $r$.

There is a continuum of buyers (or customers) of measure 1 and a continuum of sellers (or suppliers) of measure $\sigma$. Buyers want to trade with sellers. Each buyer gets the urge to consume at random times which arrive at Poisson rate $\alpha$ and when this occurs he has to purchase the commodity.\(^5\)

The price of the commodity is exogenously fixed at $p$ but the quantity and/or purity that is sold may vary. To keep the analysis simple, we restrict attention to pure quantity=quantity*purity as a measure of the utility that the buyer receives from a transaction and ignore what it consists of (e.g. the same pure quantity can be the result of a large quantity of low purity or a small quantity of high purity). From now on we refer to the quality of the good transacted instead of its “pure quantity” and we denote it by $q$.

\(^5\)Implicitly the buyer gets infinite disutility if he does not consume when he gets the urge to do so. Alternatively, he could draw the cost of not consuming from some distribution. Then the cutoff cost (below which he would choose to forgo consumption) would be an equilibrium object depending on market conditions. Furthermore, we could have two types of users: regular and infrequent users, characterized by their $\alpha$. 

10
A buyer cannot evaluate the quality of what is on sale at the time that he meets with a seller. After buying and consuming the good, however, the quality of the purchase is perfectly revealed. This is important because it affects the buyer’s decision of whether to return to the same seller the next time that he wants to consume.

The market is characterized by search frictions in the sense that there is no central marketplace where all agents can meet to trade. Rather, buyers and sellers have to trade bilaterally. A buyer can be in either of two states: matched, which means that he has a regular supplier, or unmatched. An unmatched buyer has to search in the market at random, incurring utility cost of search, $s$. A matched buyer also has the option of visiting his regular supplier, which does not entail any cost. However, there is a probability $\gamma$ that the regular supplier is unavailable, in which case the buyer has to search at random and pay $s$. The transition between these two states takes place after the trading is done.

We now detail the transitions between the two states. An unmatched buyer decides whether to match with a seller after consuming his good. If this occurs, the seller becomes his regular supplier. A matched buyer decides whether to switch sellers if his regular supplier was unavailable and he sampled a new seller at random. A match between a buyer and a seller is exogenously destroyed at rate $\delta$. In this event, the buyer becomes unmatched. The substantive implication of this discussion is that a buyer can only be matched with one seller at a time and there is no recall: once a match has been broken, the buyer cannot find that seller again.

The seller chooses the quality level that he supplies to the buyers that visit him. Providing with higher quality is more costly but it may make buyers more likely to come back. The main assumption on sellers’ behavior is that, once they decide on the quality level that they offer, they commit to their decision forever. That is, each seller supplies the same quality to all customers at all times and, as a result, a buyer knows the quality that he will receive from a particular seller once he has sampled from him.

We will look for steady state equilibria.

### 3.2 The Buyers’ Decision Problem

We first consider the buyers’ problem, taking the sellers’ actions as exogenous.

Let $F$ denote an arbitrary distribution of qualities in the market with support in $[0, q]$. The state variables for a buyer is whether he is matched and, if so, what is the quality that he receives from his regular supplier. Let $\bar{V}$ denote the value function of a buyer who is unmatched. Let $V(q)$ denote the value of being matched with a seller who offers quality $q$. 


The value functions in flow terms are given by the following asset pricing equations:

\[
\begin{align*}
    r \bar{V} &= \alpha \left[ -s + \int_{0}^{\bar{q}} \left[ \tilde{q} + \max\{V(\tilde{q}), \bar{V}\} - \bar{V} \right] dF(\tilde{q}) - p \right] \tag{1} \\
    r V(q) &= \alpha \left[ (1 - \gamma)q + \gamma \int_{0}^{\bar{q}} \left[ -s + \tilde{q} + \max\{V(\tilde{q}), V(q)\} - V(q) \right] dF(\tilde{q}) - p \right] \\
    &\quad + \delta (\bar{V} - V(q)) \tag{2}
\end{align*}
\]

The interpretation is as follows. Consider equation (1) first. At rate \( \alpha \) the buyer gets the urge to consume. When this happens, he samples a seller at random, incurs the cost of search \( s \), pays \( p \), and consumes. The instantaneous utility that he receives from consuming is a random draw from the distribution of qualities, \( F \). After consuming, the buyer decides whether to keep this seller as his regular supplier, which yields a “capital gain” of \( V(\tilde{q}) - \bar{V} \), or to remain unmatched, in which case there is no change in his value. Equation (2) is similar. Again, at rate \( \alpha \) the buyer wants to consume. With probability \( 1 - \gamma \) his supplier is available and the buyer receives quality \( q \). With probability \( \gamma \) the regular seller is unavailable and the buyer has to search in the market. As a result, he incurs cost \( s \) and he makes a random draw from \( F \). In both cases he pays \( p \) to the seller that he transacts with. The only difference between searching when matched or unmatched is that when the buyer is matched he compares the new seller with his regular supplier when deciding whether to stay with the new draw. Therefore the capital gain of switching to the new seller is \( V(\tilde{q}) - V(q) \). Last, at rate \( \delta \) the match is destroyed and the buyer becomes unmatched leading to a capital loss of \( \bar{V} - V(q) \).

Since \( V(q) \) is strictly increasing in \( q \) and \( \bar{V} \) is independent of \( q \) there is a unique reservation quality \( R \). An unmatched buyer who samples a seller offering quality \( q \geq R \) will choose to match with the current seller, while if \( q < R \) he will remain unmatched.\(^6\) Furthermore, a matched buyer switches suppliers if and only if the new seller offers a higher quality.

The rest of the analysis of the buyers’ behavior centers on characterizing \( R \) as a function of the (still) exogenous distribution \( F \). Using the reservation properties that we derived, we can now simplify the two equations above in the following way (defining \( E(\tilde{q}) \equiv \int_{0}^{\bar{q}} \tilde{q} dF(\tilde{q}) \))

\[
\begin{align*}
    r \bar{V} &= \alpha \left[ -s + E(\tilde{q}) + \int_{R}^{\bar{q}} (V(\tilde{q}) - V) \ dF(\tilde{q}) \right] \tag{3} \\
    r V(q) &= \alpha \left[ (1 - \gamma)q + \gamma[-s + E(\tilde{q}) + \int_{q}^{\bar{q}} (V(\tilde{q}) - V(q)) dF(\tilde{q})] + \delta (\bar{V} - V(q)) \right. \tag{4}
\end{align*}
\]

These two equations can be manipulated to eliminate the value function \( V \), thus express-
ing $R$ as a function of $F$ alone.

**Lemma 1** The reservation quality is given by

$$R(F) = \max\{\hat{R}, 0\},$$

where $\hat{R}$ is defined by

$$\hat{R} = -s + E(q) + \alpha (1 - \gamma) \int_{\hat{R}}^{\overline{q}} \frac{1 - F(q)}{r + \delta + \alpha \gamma (1 - F(q))} dq.$$  \hspace{1cm} (6)

**Proof.** To find the reservation quality we first derive the quality level $\hat{R}$ such that $V(\hat{R}) = \overline{V}$. Since quality has to be non-negative, the reservation quality is given by the maximum of $\hat{R}$ and zero. $\hat{R}$ could potentially be negative due to the search cost of being unmatched $s$ which gives a bonus to matching regardless of quality.

We can evaluate (4) at $q = \hat{R}$ and equate it with (3) to get

$$R + s = E(\bar{q}) + \int_{\hat{R}}^{\overline{q}} (V(\bar{q}) - \overline{V}) dF(\bar{q}).$$

(7)

We calculate $V'(q)$ in order to integrate by parts the integral on the right hand side. Differentiating equation (4) with respect to $q$ yields

$$r V'(q) = \frac{\alpha (1 - \gamma)}{r + \delta + \alpha \gamma (1 - F(q))}.$$  \hspace{1cm} (8)

Now, integrating by parts

$$\int_{\hat{R}}^{\overline{q}} (V(\bar{q}) - \overline{V}) dF(\bar{q}) = \int_{\hat{R}}^{\overline{q}} V(\bar{q}) dF(\bar{q}) - (1 - F(R))\overline{V}$$

$$= V(\overline{q}) - V(R) F(R) - \int_{\hat{R}}^{\overline{q}} F(\bar{q}) V'(\bar{q}) d\bar{q} - (1 - F(R))V(R)$$

$$= \int_{\hat{R}}^{\overline{q}} (1 - F(\bar{q})) V'(\bar{q}) d\bar{q},$$  \hspace{1cm} (9)

since $V(\overline{q}) - V(R) = \int_{\hat{R}}^{\overline{q}} V'(\bar{q}) d\bar{q}$. Substituting equations (8) and (9) into (7) yields (6). \hfill \blacksquare

### 3.3 The Sellers’ Decision Problem

We assume that sellers maximize their steady state level of profits and concentrate on the case where $r \to 0$ for simplicity. The steady state profits of a seller who chooses to offer
quality level $q$ are given by

$$\pi(q) = (p - c q) \ t(q)$$

(10)

The first terms is the seller’s margin per sale (where $c$ is the marginal cost of quality) and $t(q)$ is his steady state number of transactions. $t(q)$ depends on the the distribution of qualities, $F$ and the workers’ reservation strategy, $R$.

3.4 Steady-State Flow of Trades

In this section we characterize the equilibrium level of sales $t(q)$ of a seller who offers quality $q$ as a function of the distribution of qualities, $F$, and the buyers’ reservation quality, $R$. The only assumption that is used in this section is that the equilibrium is a steady state, and so the aggregate population statistics (number of matched and unmatched sellers, etc.) are constant over time.

Sales come from two sources: the steady state number of ‘loyal’ (or regular) customers and the random customers who sample once and may or may not become regular after consuming (if they do become regulars, they are counted as ‘loyal’ from then on). Denote the flow of regular and occasional buyers by $t_R(q)$ and $t_O$, respectively. The flow of total sales is given by

$$t(q) = t_O + t_R(q).$$

(11)

We henceforth characterize $t_R(q)$ and $t_O$.

Let $\beta = 1/\sigma$ denote the total number of buyers per seller. When deriving the transaction flow it will prove more convenient to work with the number of buyers rather than sellers. Let the steady state number of unmatched and matched buyers be given by $n$ and $\beta - n$, respectively. In steady state, the flows of buyers from the matched state to the unmatched state and vice versa equal each other. An unmatched buyer becomes matched after sampling a seller who offers above-reservation quality which occurs at rate $\alpha (1 - F(R))$. A matched buyer becomes unmatched when his match is exogenously destroyed which occurs at rate $\delta$. As a result, in steady state the following holds:

$$n \alpha (1 - F(R)) = (\beta - n) \ \delta.$$

Isolating $n$ yields

$$n = \frac{\beta}{1 + (\alpha/\delta) (1 - F(R))}.$$
The flow of occasional customers consists of unmatched buyers and of matched buyers whose regular supplier is unavailable. Therefore

\[ t_o = \alpha \gamma (\beta - n) + n = \alpha \beta \frac{\delta + \alpha \gamma (1 - F(R))}{\delta + \alpha (1 - F(R))}. \]

Characterizing the flow of regular sales is a bit more involved because we first need to derive a seller’s number of regular buyers which depends on the quality offered. Let \( l(q) \) denote the number of regular customers of a seller offering \( q \). The flow of regular trades for a seller offering quality \( q \geq R \) is given by

\[ t_R(q) = \alpha (1 - \gamma) \, l(q) \]

We proceed to characterize \( l(q) \). It is immediate that \( l(q) = 0 \) when \( q < R \). To describe \( l(q) \) for \( q \geq R \), let \( G \) denote the distribution of qualities that matched buyers receive. \( G \) first order stochastically dominates the distribution of offered qualities because a matched buyer moves to higher qualities over time. The number of matched buyers receiving quality up to \( q \) is given by \((\beta - n) \, G(q)\). The flow into this group comes from the \( n \) unmatched buyers who got the urge to consume and drew a quality level that they chose to keep (i.e. above \( R \)) but which is no greater than \( q \). Note that there are also movements within this group (i.e. from some \( q_1 \) to \( q_2 \) with \( R \leq q_1 < q_2 \leq q \)) but these do not affect the flow in question. Buyers flow out of this group either because their match is exogenously destroyed or because their regular seller was unavailable when wanted to consume and they sampled a quality level higher than \( q \) which made them switch.

Equating the flows and solving for \( G(q) \) yields

\[
n \cdot \alpha \left[ F(q) - F(R) \right] = (\beta - n) \, G(q) \left[ \delta + \alpha \gamma (1 - F(q)) \right] \\
\Rightarrow G(q) = \frac{F(q) - F(R)}{(1 - F(R)) \left( 1 + (\alpha/\delta) \, \gamma (1 - F(q)) \right)}
\]

for \( q \geq R \), and \( G(q) = 0 \) otherwise.

The average number of buyers matched to a seller offering quality level \( q \) is given by

\[
l(q) = \lim_{\epsilon \to 0} (\beta - n) \frac{G(q) - G(q - \epsilon)}{F(q) - F(q - \epsilon)} = (\beta - n) \frac{G(q)}{F(q)}
\]

(we assume here, and later verify, that \( F \) is differentiable for \( q > R \)). It is a matter of algebra
to arrive at
\[
q = \frac{\alpha \beta}{[\delta + \alpha \gamma (1 - F(q))]^2 \delta + \alpha (1 - F(R))}
\]

The following lemma summarizes our results.

**Lemma 2** The steady state level of transactions of a seller offering quality \( q \) is given by
\[
t(q) = \alpha \beta \frac{\delta + \alpha \gamma (1 - F(R))}{\delta + \alpha (1 - F(R))} [1 + \frac{\alpha (1 - \gamma)}{\delta + \alpha \gamma (1 - F(R))]^2}, \text{ when } q \geq R, \tag{12}
\]
\[
t(q) = \alpha \beta \frac{\delta + \alpha \gamma (1 - F(R))}{\delta + \alpha (1 - F(R))}, \text{ when } q < R. \tag{13}
\]

### 3.5 A Preliminary Result

Our first substantive result is that changing the number of sellers in the market changes the equilibrium level of profits but has no further effect on the distribution of quality or the buyers’ reservation.

It is clear from equations (12) and (13) that the buyer-seller ratio enters profits multiplicatively, scaling the total number of transactions, \( t(q) \). Furthermore, \( \beta \) does not enter the decision problem of the agents anywhere else. As a result, we can rearrange (10), recalling that the number of sellers is \( \sigma = 1/\beta \), to get
\[
\sigma \pi(q) = (p - c q) \tilde{t}(q) \tag{14}
\]

where \( \tilde{t}(q) \equiv t(q)/\sigma \) does not depend on the buyer-seller ratio. The right-hand side gives the total industry profits which do not depend on the number of buyers or sellers in the market. The left-hand side makes clear that as the number of sellers increases, per-seller profits drop by the same factor. We have the following result.

**Proposition 3** (Total industry profits invariant to number of sellers) Increasing the number of sellers does not affect the agents’ equilibrium behavior. When the number of sellers increases by a factor \( \tau \) each seller’s equilibrium profit decreases by a factor \( 1/\tau \).

From now on we assume \( \beta = \sigma = 1 \).

### 3.6 Equilibrium Definition

**Definition 4** An equilibrium is a triple \( \{R, F, \bar{\pi}\} \) such that the following conditions hold:
1. Buyer optimization: \( R = R(F) \) where \( R(F) \) is defined in (5).

2. Seller optimization: \( \pi(q) = \bar{\pi} \forall q \in \text{supp} F \) and \( \pi(q) < \bar{\pi} \forall q \not\in \text{supp} F \).

4 Equilibrium Quality Dispersion and Its Comparative Statics

To characterize \( F \), we use the fact that in equilibrium all qualities that are offered need to yield the same steady state profits. Recall that increasing the quality offered affects profits in two ways: it reduces the margin per transaction and it increases the number of transactions by raising the steady state number of regular customers. The number of customers increases in \( q \) because a higher-quality seller has more competitors from whom to poach customers (higher inflow) and because there are fewer sellers that can poach his own customers (lower outflow).

Recall that \( \bar{q} \) is the supremum of the support of \( F \). We start by deriving a set of properties that \( F \) satisfies in equilibrium.

**Proposition 5** The following properties hold in equilibrium:

(i) If \( q \in \text{supp} F \) then either \( q = 0 \) or \( q \geq R \).

(ii) \( F \) has no mass point in its positive part.

(iii) The positive part of the support of \( F \) is connected and it is given by \([R, \bar{q}]\).

(iv) \( F \) exhibits quality dispersion.

**Proof.** A seller who offers \( q \in [0, R) \) has no regular customers. As result \( t(q) = t_O \) for all \( q \in [0, R) \) and any positive quality is dominated by \( q = 0 \). Therefore if any quality is offered below \( R \), it must be zero quality.

Suppose that a discrete mass of sellers offers quality \( q^* \geq R \). A seller who offers \( q^* + \epsilon \) can poach customers from the whole mass of suppliers offering exactly \( q^* \), leading to a discrete increase in the inflow of buyers. Such a seller would thus get discretely more steady state sales than any seller offering \( q^* \), with only negligible additional cost. This means that \( \pi(q^* + \epsilon) > \pi(q^*) \) which cannot hold in equilibrium.

Suppose there is a gap in the support of \( F \) between \( q_1 \) and \( q_2 \), where \( R \leq q_1 < q_2 \leq \bar{q} \). The sellers offering \( q_1 \) and \( q_2 \) have exactly the same number of regular customers since they poach from the same set of competitors and hence \( t(q_1) = t(q_2) \). Since it is cheaper to offer \( q_1 \) we have \( \pi(q_1) > \pi(q_2) \), which cannot be part of an equilibrium.

Let \( \underline{q} \) be the lowest positive quality on offer. Then \( t(R) = t(\underline{q}) \) which means that in equilibrium \( \underline{q} = R \).
Suppose that $F(0) = 1$. Then $R = 0$ and the no-mass point argument yields a contradiction. Therefore, any $F$ exhibits quality dispersion in equilibrium.

The proposition above shows that there are two distinct parts to the distribution of offered qualities: a mass point at zero and a continuous part starting at the buyers’ reservation $R$.

Depending on the parameter values, in equilibrium there may be a fraction of sellers who offer zero quality. In the appendix we show that in equilibrium a positive measure of sellers offer zero quality if and only if

$$\frac{p/c}{s} > \frac{1}{k_1^2} + \frac{1}{k_0 - k_1} \left(1 + \frac{1}{k_1}\right)^2,$$

In this section we characterize such equilibria. The quality distribution is denoted by $F$. The maximal quality offered in equilibrium is denoted by $\bar{q}$. The lowest positive quality offered in equilibrium is denoted by $R$.

### 4.1 Maximal quality offered

The seller who offers zero makes profits of

$$\pi(0) = p \alpha O = p \alpha b \frac{\delta + \alpha \gamma L}{\delta + \alpha L}$$

The seller who offers $q$ makes profits of

$$\pi(q) = (p - c \bar{q}) (\alpha_O + \alpha_R(\bar{q})) = (p - c \bar{q}) \alpha b \left(1 + (1 - \gamma) (\alpha/\delta)\right) \frac{\delta + \alpha \gamma L}{\delta + \alpha L}$$

Equating the two profits and solving for $\bar{q}$ yields

$$\bar{q} = \frac{p}{c} \cdot \frac{\alpha(1 - \gamma)}{\delta + \alpha(1 - \gamma)}$$

Since $\bar{q}$ is increasing in $\alpha(1 - \gamma)$, we have the following lemma.

**Lemma 6** The maximal quality offered is increasing in $\alpha, p$ and is decreasing in $\gamma, \delta, c$.

### 4.2 Solving for the distribution of quality

The profits of sellers offering 0 and $q$ are given by
π(0) = α₀ \ p
π(q) = (α₀ + α_R(q)) (p - c \ q), \text{ for } q \geq R.

Substitute for α₀ and α_R(q) and equate π(0) and π(q) in order to solve for the distribution of qualities offered in equilibrium. After some algebra, we get the following function (which, for convenience, is defined on the entire $\mathbb{R}_+$):

$$F(q) = 1 + \frac{\delta}{\alpha \gamma} \left(1 - \sqrt{\frac{\alpha (1 - \gamma)}{\delta}} \sqrt{\frac{p - q}{c q}}\right). \quad (15)$$

On the interval $[R, \bar{q}]$, the c.d.f. of qualities offered in equilibrium coincides with the function $F(q)$. Outside of that interval, only zero quality is offered. Formally,

$$F(q) = \begin{cases} F(q) & \text{for } q \in [R, \bar{q}], \\ F(R) & \text{for } q \in [0, R]. \end{cases}$$

Differentiating this expression we get the density of the qualities offered in equilibrium,

$$f(q) = \frac{1}{2} \sqrt{\frac{\delta (1 - \gamma)}{\alpha \gamma^2}} \cdot \sqrt{\frac{(\frac{p}{c})^2}{(\frac{p}{c} - q) q^3}} \text{ for } q \in [R, \bar{q}]. \quad (16)$$

**Proposition 7** The density distribution of sellers offering quality $q$ is decreasing in $q$ if $\alpha (1 - \gamma) \leq 3 \delta$; if the opposite inequality holds, then the density is U-shaped in $q$.

**Proof.** The function $f(q)$ defined in expression (16) is U-shaped with a minimum at $q = \frac{3 p}{4 c}$. The support of $f$, of course, is bounded above by $\bar{q}$. So if $\bar{q} > \frac{3 p}{4 c}$, that is, if $\alpha (1 - \gamma) > 3 \delta$, then $f$ is u-shaped; otherwise, $f$ is monotonically decreasing on its support. ■
4.3 Uniqueness of $R$

If $R$ is part of an equilibrium it solves equation (6). Rewriting $E(\tilde{q})$ and rearranging yields

$$R F(R) + s = \int_{\tilde{q}} R (1 - F(\tilde{q})) d\tilde{q} + \alpha (1 - \gamma) \int_{\tilde{q}} \frac{1 - F(\tilde{q})}{\delta + \alpha \gamma (1 - F(\tilde{q}))} d\tilde{q}$$

As characterized in (15), the expression for $F$ does not depend on $R$. Thus the LHS of (17) is increasing in $R$, and the RHS is decreasing in $R$. So there is at most a one $R$ that is compatible with equilibrium.

**Proposition 8** The equilibrium is unique.

4.4 Comparative statics with respect to $s$

As characterized in (15), the expression for $F$ does not depend on $s$. Thus in equation (17) $s$ enters the LHS only, and then only as an additive constant. So, as $s$ increases to $s'$ the equilibrium $R$ decreases to $R'$. Moreover, since $F(q)$ does not depend on $s$ or $R$, as $s$ increases the shape of $F(q)$ is unchanged for $q > R$. Thus, an increase in $s$ results in a stochastically dominant shift of the distribution $F$. These observations can be rephrased as follows.

**Proposition 9** As the consumer’s search cost increases, the average quality of drugs offered by sellers increases. After the change, some sellers who were previously offering zero quality now offer a positive quality. These sellers will offer a previously unavailable range of quality, which is just inferior to the lowest quality previously offered. The distribution of positive qualities previously offered by sellers is unchanged.

It should be noted that although the quality of drugs offered improves when search costs increase, consumers are not necessarily better off, due to the direct effect of the increase in $s$.

4.5 Comparative statics with respect to $c, p$

The stylized fact (according to Reuter, personal communication) is that quality traded decreases in response to interdiction at the source. We mimick the effect of interdiction at the source by increasing $c$ in our model. What happens to the distribution of qualities traded? To answer this question, we notice that from equation (15) we have

$$\frac{\partial F(q)}{\partial c} > 0.$$
So a small increase in $c$ increases $F$ in the interval $(R, \bar{q}]$. This would suggest the possibility of a stochastic dominance result, but actually we are only able to prove a weaker property, namely, that all the quantiles of $F$ which lie above $R$ (including the median, if $F(R) < 1/2$) become smaller in response to a small increase in $c$.

**Proposition 10** Let $\phi$ denote any quantile in the positive part of the support of the quality distribution. Now increase $c$ (or, equivalently, decrease $p$), and let $\tilde{\phi}$ denote the same quantile in the new equilibrium distribution. Then $\tilde{\phi} < \phi$.

**Proof.** Let a tilde denote equilibrium objects after the increase in $c$. Fix $y > F(0)$ and denote the corresponding quantiles before and after the increase in $c$ by $\phi = F^{-1}(y)$ and $\tilde{\phi} = \tilde{F}^{-1}(y)$. We can write

$$\tilde{\phi} = \tilde{F}^{-1}(y) \leq \tilde{F}^{-1}(y) < F^{-1}(y) = F^{-1}(y) = \phi$$

where the weak inequality reflects the definition of $\tilde{F}(q) = \max \left[ \tilde{F}(q), F(\tilde{R}) \right]$; the strict inequality comes from $\partial F(q)/\partial c > 0$; and the second-to-last equality follows from $y > F(0)$.

This result does not imply a stochastic dominance shift of $F$ in response to an increase in $c$, because the mass of sellers offering zero quality might actually decrease. In fact, $R$ decreases in response to the increase in $F(\cdot)$. To see this, observe that $R$ depends on $c$ only through the increase in $F(\cdot)$ in equation (17). The RHS of (17) is monotonic in $1 - F$ for fixed $R$, so as $F$ increases the RHS goes down. Meanwhile, for fixed $R$ the LHS of (17) goes up. So $R$ needs to decrease to maintain equation (17).

### 5 Testable Implications of the model

#### 5.1 Quality dispersion and rip-offs

Our model predicts rip-offs (in the language of Section 4, $F(R) > 0$), a gap in the quality offered above zero quality, and dispersion in the set of positive qualities offered. A graphical rendition of the distribution of qualities traded in the equilibrium of our model is presented in the following figure.
Quality dispersion predicted from the model

Evidence available in the STRIDE data is apparently consistent with our predictions concerning quality dispersion. An example of the quality dispersion seen in the data is reported in the next figure, where we show the quality (purity x quantity) of heroin involved in transactions in New York costing $100 from 1995 to 1997.\(^8\)

The data appear broadly consistent with our predictions. There is a large fraction of rip-offs, with over 12% of the transactions involving sellers who sold heroin consisting of zero purity. Even more striking is the gap between observed between zero potency observations (the pure rip offs) and the low quality sellers, as predicted by our model. Finally, the STRIDE data support the notion of purity dispersion in the market, as no single modal level of quality

\(^8\)The presence of price and purity dispersion in the market for illicit drugs has been noted before by Caulkins (1995) and Reuter and Caulkins (2004). Reuter and Caulkins (2004) examine 14 years of purchase data for cocaine and heroin from STRIDE and find extremely high price and purity dispersion. Their paper does not offer a formal theoretical model that generates dispersion, however.
emerges. Instead, we get dispersion in the price/quality ratio consistent with other models of search (Burdett and Mortensen, 1998).9

5.2 Loyalty of frequent and occasional buyers

Are frequent buyers more loyal to their supplier? In our model the frequency with which buyers purchase is captured by the parameter \( \alpha \). Although we do not have differences in \( \alpha \) across buyers in our model, we can conjecture about the behavior of buyers with different \( \alpha \)'s. if we introduced in our model a very frequent buyer (\( \alpha \) close to \( \infty \)) that buyer would quickly become matched with a seller, and so would not search in equilibrium. Conversely, if we introduced in the model a very infrequent buyer (\( \alpha \) close to zero) that buyer typically would take a long time before getting hooked up with a regular seller, and until he did he would search by repeatedly sampling from opportunistic sellers. Thus the infrequent buyer would be expected to be less loyal to his sellers because the initial sampling phase would last very long. In addition, once sellers and buyers are matched, the frequent buyer would be expected to be matched to a better quality seller, and so to be less likely to switch sellers when matches separate.10 In sum, our model suggests that frequent buyers should be more likely to be loyal to their sellers.

This prediction is consistent with the evidence from the ADAM data. The next table displays a stochastic dominance relationship between the source of frequent and infrequent buyers.11

---

9 Heterogeneity in people’s willingness to search and search costs could also generate a range of transaction prices for even a homogeneous good (Stigler, 1961), and the same is true for drug markets (Reuter and Caulkins, 2004). This effect could partly account for the quality dispersion we observe. In our model, all the dispersion comes from the different length in the relationship between buyers and sellers. Buyers who have been around for a long time tend to be matched with better sellers. Even with the assumption that all sellers make the same level of profit, the dispersion of quality is sustained because some sellers offer low (even zero) quality, making a large profit per transaction but with relatively few repeat customers, and other sellers offer better quality, getting lower profit per transaction but more repeat customers.

10 A more frequent consumer (higher \( \alpha \)) has a higher reservation quality, ie is pickier with his suppliers, because he consumes more per unit of time once he settles with some seller. As a result, the average matched frequent consumer enjoys higher quality than the average matched casual consumer. Therefore, when forced to search, the casual consumer is more likely to find a better deal than he currently enjoys and switch.

11 For the purpose of this table a frequent buyer is defined as a buyer who purchased more than 28 times in the past 30 days. A Pearson chi-square test rejects the hypothesis that the distribution of frequent and infrequent buyers are the same (p=0.000).
<table>
<thead>
<tr>
<th>Source from whom you bought heroin</th>
<th>Infrequent buyers</th>
<th>Frequent buyers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular dealer</td>
<td>290</td>
<td>468</td>
</tr>
<tr>
<td></td>
<td>58.3%</td>
<td>76%</td>
</tr>
<tr>
<td>Occasional dealer</td>
<td>162</td>
<td>117</td>
</tr>
<tr>
<td></td>
<td>32.6%</td>
<td>19%</td>
</tr>
<tr>
<td>New dealer</td>
<td>45</td>
<td>31</td>
</tr>
<tr>
<td></td>
<td>9%</td>
<td>5%</td>
</tr>
<tr>
<td>Total</td>
<td>497</td>
<td>616</td>
</tr>
<tr>
<td></td>
<td>100%</td>
<td>100%</td>
</tr>
</tbody>
</table>

This evidence is consistent with our model.

5.3 Larger transaction yield higher quality distributions

The figure presented in the previous section allows us to compare the distribution of qualities traded at different amounts. Comparing across the three trade values, we see that each quartile in a given trade is smaller than the same quartile in a larger trade. This is consistent with the comparative static developed in Section 4.5 (comparative static with respect to $p$).

6 Conclusions

(PRELIMINARY)

In this paper we offer a theoretical framework that incorporates some key elements of the retail markets for illicit drugs. A prominent role is played by the ability of sellers to cut the drugs without being immediately caught by the customers. This moral hazard, in our model, puts the retail drugs market at risk of collapse for “overcutting.” Consistent with this view, the data reveal a large amount of “scam” transactions. The countervailing force that supports trade in our model is the presence of repeated interactions, which also is found in the data.

If it’s true that long-term relationships contribute to the viability of the retail drugs market—and we believe the evidence suggests it is true—then a number of important consequences follow. First, increasing enforcement on buyers (greater risk of being caught, greater penalties) may lead to unintended consequences: buyers will try to avoid these “search costs” by sticking with their regular sellers more, thus boosting the length of the relationship. In equilibrium, this leads sellers to increase the quality of the drugs they sell. Second, only enforcement that permanently breaks up relationships between buyers and sellers has a positive
impact on the market (reduced quality). “Temporary” enforcement induces buyers to sample from other sellers, which increases the competition among sellers and may, perversely, lead sellers to offer better quality. Findings from this model suggest that increased enforcement on buyers and sellers at the retail level may have contributed in part to the long term decline in the average price per expected pure gram in cocaine and heroin in the US (Caulkins et al., 2004), though more evidence needs to be marshalled before drawing conclusions concerning the effect of enforcement.

A message from this paper is that we should pay more attention to the length of the relationship between buyers and sellers as a measure of the effectiveness of local enforcement. A particularly interesting measure, in light of our analysis, is the correlation between purity of drugs transacted and length of the buyer-seller relationship—a point on which unfortunately there is scant empirical evidence to date.
References


A Equilibrium for Arbitrary Number of Opportunistic Sellers

We have previously shown that $F$ consists of a continuous part on $[R, q]$ and potentially a mass point at 0, if $R > 0$. We proceed to characterize $F$ for an arbitrary number of opportunistic sellers $F(0) \in [0, 1)$. Subsequently we characterize $F(0)$.

To characterize $F$, we use the equilibrium condition that all sellers make the same steady state profits. There are two steps to characterizing the distribution of qualities. We first fix an arbitrary $R$ and derive $F$ such that all sellers make the same profits. We then put our solution together with equation (6) which describes the optimal buyer behavior for an arbitrary $F$. This gives us the unique fixed point in the strategies of the agents.

Proposition 11 (Distribution of Qualities for arbitrary $R$) The distribution of qualities offered in the market is given by

$$F(q) = 1 + \frac{1}{k_1} \left[ 1 - \sqrt{h(q)} \right]$$

where

$$h(q) = \frac{p/c - q}{(q - R)/(k_0 - k_1) + (p/c - R)/(1 + k_1 L)^2}$$

Proof. Fix buyers’ behavior. In equilibrium $\bar{\pi} = \pi(R) = (p - R c) t(R)$ and this expression only depends on model parameters and $R$. Taking $F(0) = 1 - L$ and $R$ as given we want to construct $F$ such that

$$\pi(R) = \pi(q) \iff (p - c R) [\alpha_R(q) + \alpha_O] = (p - c q) [\alpha_R(q) + \alpha_O] \iff \alpha_R(q) = \frac{p - c R}{p - c q} (\alpha_O + \alpha_R(R)) - \alpha_O$$

Recalling that $\alpha_O = \alpha b \frac{1 + k_1 L}{1 + k_0 L}$ we can use equations (??) and (??) to construct $F$ such that profits are equalized. After some algebra, equation (20) becomes

$$F(q) = 1 + \frac{1}{k_1} - \frac{1}{k_1} \frac{p/c - q}{(q - R)/(k_0 - k_1) + (p/c - R)/(1 + k_1 L)^2}$$

$$\equiv 1 + \frac{1}{k_1} - \frac{1}{k_1} \sqrt{h(q)}$$

\[\text{\textsuperscript{12}}\] It is easy to show that if $F(0) = 1$ then $R = 0$ which contradicts the no mass-point lemma above. Therefore, $F(0) \in [0, 1)$.
As a result we now have an expression for the distribution of qualities that are offered in the market given some arbitrary \( R \) and \( L \). Furthermore, we can solve for the highest quality on the market by setting \( F(\bar{q}) = 1 \) to get

\[
\bar{q} = \frac{k_0 - k_1}{k_0 - k_1 + 1} \left[ \frac{p}{c} + \frac{R}{k_0 - k_1} - \frac{p/c - R}{(1+k_1L)^2} \right]
\] (23)

The next step is to integrate our expression for \( F \) into (6) which gave \( R \) as a function of \( F \). In this way, we will end with the equilibrium \( R \) depending only on model parameters and \( L \). The next proposition states the result.

**Proposition 12** Fix the proportion of opportunistic sellers to some \( F(0) \in [0,1) \). The equilibrium reservation quality is given by

\[
R = \frac{p/c \ (k_0 - k_1) \ (k_1 L)^2 - s \ (1 + k_1 L)^2}{(1 - L) \ (1 + k_1 L)^2 + (k_0 - k_1) \ (k_1 L)^2}, \quad \text{if} \quad \frac{p/c}{s} \geq \frac{1}{k_0 - k_1} \ (1 + \frac{1}{k_1 L})^2 \quad (24)
\]

\[
R = 0, \quad \text{otherwise.} \quad (25)
\]

**Proof.** We first derive \( \dot{R} \) and then check whether it is greater than zero.

Note that \( F(q) = 1 - L \ \forall \ q \leq R \). This means that

\[
E[\bar{q}] = \int_0^\bar{q} \bar{q} \ dF(\bar{q})
= \int_R^\bar{q} \bar{q} \ dF(\bar{q})
= (1 - F(R)) \ R + \int_R^\bar{q} (1 - F(\bar{q})) \ d\bar{q}
\]

where the last equality is a direct result of integration by parts. We can therefore rewrite (6) as (recalling that \( k_0 = \alpha/\delta, k_1 = k_0 \gamma \))

\[
\dot{R} = -s + L \ \dot{R} + \int_\bar{q}^\pi (1 - F(\bar{q})) \ d\bar{q} + (k_0 - k_1) \int_\bar{q}^\pi \frac{1 - F(\bar{q})}{1 + k_1 (1 - F(\bar{q}))} \ d\bar{q}
\]

\[
\dot{R} \ (1 - L) = -s + \int_\bar{q}^\pi \frac{(k_0 - k_1) \ (1 - F(\bar{q}))}{1 + k_1 (1 - F(\bar{q}))} \ d\bar{q} \quad (26)
\]
Using (16), this expression can be simplified to

\[ k_1 [\hat{R} (1 - L) + s] - (\bar{q} - \hat{R}) (k_0 - k_1 - 1) = \int_{\hat{R}}^{\bar{q}} \left[ \sqrt{h(q)} - \frac{k_0 - k_1}{\sqrt{h(q)}} \right] dq \]  

(27)

where the key is to evaluate the following integral:

\[ I = \int_{\hat{R}}^{\bar{q}} \left[ \sqrt{h(q)} - \frac{k_0 - k_1}{\sqrt{h(q)}} \right] dq \]  

(28)

This integration is somewhat complicated. In order to solve it note that the expression inside the square root takes the form \( h(q) = \frac{a_1 - a}{q} + a_2 + a_3 \) where \( a_1, a_2, a_3 > 0 \). Then we can change variables successively and we show in the appendix the solution yields equation (24). It is straightforward to show that \( \hat{R} < p/c \). The last thing to check is that \( \hat{R} \geq 0 \) which yields the condition following equation (24).

The last step is to determine the equilibrium number of opportunistic sellers. We pin it down by equalizing profits across opportunistic and non-opportunistic sellers. The following proposition states the result.

**Proposition 13** A positive measure of sellers offer zero quality only if

\[ \frac{p/c}{s} > \frac{1}{k_1^2} \frac{1}{k_0 - k_1} \left( 1 + \frac{1}{k_1} \right)^2, \]  

(29)

and in that case \( F(0) \) is uniquely defined. Otherwise, \( F(0) = 0 \).

**Proof.** Let \( \pi_O = \pi(0) \) and \( \pi_L = \pi(q) \), \( q \in [R, \bar{q}] \). Then,

\[ \pi_O = \alpha_O p \]  

(30)

\[ \pi_L = (p - c R) (\alpha_O + \alpha_R(R)) \]  

(31)

Let \( \Delta \pi(L) \equiv \pi_L(R) - \pi_O \) where the profits are determined for a fixed \( L \). The goal is to characterize \( L^* \) such that \( \Delta \pi(L^*) = 0 \). Algebra shows that

\[ \Delta \pi(L) = b \alpha c \left( \frac{1 + k_1 L}{1 + k_0 L} \right) \left[ \frac{(p/c - R)}{(1 + k_1 L)^2} - R \right] \]

\[ = \frac{b \alpha c (k_0 - k_1) (1 + k_1 L) s}{(1 + k_0 L) [(1 - L) (1 + k_1 L)^2 + (k_0 - k_1) (k_1 L)^2]} N(L), \]  

(32)
where

\[
N(L) = \frac{p/c}{s} (1 - L - (k_1 L)^2) + \frac{(1 + k_1 L)^2}{k_0 - k_1} + 1
\]  

(33)

Observe that equation (32) is equal to zero only if \(N(L) = 0\). Furthermore, \(N(L)\) is a quadratic equation in \(L\) and

\[
N(0) = \frac{p/c}{s} + \frac{1}{k_0 - k_1} + 1 > 0
\]

\[
N(1) = -\frac{p/c}{s} k_1^2 + \frac{1 + k_1^2}{k_0 - k_1} + 1
\]

Therefore, \(N(1) < 0\) is necessary and sufficient for a unique solution which is the same as equation (29). In that case the number of opportunistic sellers is characterized by one of the roots of the quadratic.

If \(N(1) > 0\) then there it is easy to show that there is no root and the non-opportunistic sellers always make higher steady state profits. As a result, \(F(0) = 0\). \(\blacksquare\)

**B Solving for lowest positive quality offered**

We want to solve in closed form for the RHS of (17). Let \(k_0 = \alpha/\delta\) and \(k_1 = \gamma/\delta\).

\[
\int_R^q (1 - F(q)) \, dq + \frac{k_0 - k_1}{k_1} \int_R^q \frac{k_1 (1 - F(q))}{1 + k_1 (1 - F(q))} \, dq
\]

\[
= \frac{1}{k_1} \int_R^q (\sqrt{k_0 - k_1} \sqrt{\frac{p/c - q}{q} - 1}) \, dq + \frac{k_0 - k_1}{k_1} \int_R^q \frac{\sqrt{k_0 - k_1} \sqrt{\frac{p/c - q}{q} - 1}}{\sqrt{k_0 - k_1} \sqrt{\frac{p/c - q}{q}}} \, dq
\]

\[
= \frac{1}{k_1} \int_R^q (\sqrt{k_0 - k_1} \sqrt{\frac{p/c - q}{q} - 1}) \, dq + \frac{k_0 - k_1}{k_1} \int_R^q (1 - \frac{1}{\sqrt{k_0 - k_1} \sqrt{\frac{p/c - q}{q}}}) \, dq
\]

\[
= \frac{1}{k_1} \{ (\bar{q} - R) (-1 + k_0 - k_1) + \sqrt{k_0 - k_1} \int_R^q \sqrt{\frac{p/c - q}{q}} \, dq - \int_R^q \frac{1}{\sqrt{\frac{p/c - q}{q}}} \, dq \}
\]

Solving for these integrals is not that simple but here it is. First, do a change of variables:

\[
x = \frac{p/c - q}{q} \Rightarrow q = \frac{p/c - 1}{x}, \quad dq = -\frac{p/c - 1}{x^2} \, dx, \quad \bar{x} = \frac{p/c - R}{q}, \quad x = \frac{p/c - R}{R}.\]

Therefore the first integral can be rewritten as
\[
\int_{q}^{R} \sqrt{\frac{p/c - q}{q}} \, dq = \int_{x}^{\pi} \sqrt{x} \left( -\frac{p/c - 1}{x^2} \right) \, dx \\
= -(p/c - 1) \int_{x}^{\pi} x^{-3/4} \, dx \\
= -(p/c - 1) \left( -\frac{2}{\sqrt{x}} \right)_{x}^{\pi} \\
= 2 (p/c - 1) \left( \sqrt{\frac{q}{p/c - q}} - \sqrt{\frac{R}{p/c - R}} \right)
\]

Similarly,

\[
\int_{q}^{R} \frac{1}{\sqrt{\frac{p/c - q}{q}}} \, dq = \int_{x}^{\pi} \frac{1}{\sqrt{x}} \left( -\frac{p/c - 1}{x^2} \right) \, dx \\
= -(p/c - 1) \int_{x}^{\pi} x^{-5/4} \, dx \\
= -(p/c - 1) \left( -4 x^{-1/4} \right)_{x}^{\pi} \\
= 4 (p/c - 1) \left( \left( \frac{q}{p/c - q} \right)^{1/4} - \left( \frac{R}{p/c - R} \right)^{1/4} \right)
\]