1 The Model

- Two players are engaged in a repeated simultaneous move zero sum game.
- Player 1 (the server), can serve either left \((L)\) or right \((R)\).
- Player 2 (the receiver), can lean either left \((\ell)\) or right \((r)\).
- Assume the chance of scoring a point on serve \(t\) depends on the actions of each player in period \(t\) and the serve direction in period \(t-1\).
- Specifically, the state contingent stage game payoffs are given in Figure 1, where state \(\theta = L\) obtains if the server served left on the previous point and state \(\theta = R\) obtains if the server served right on the previous point. Note that these bonuses could be negative, the interpretation being that the receiver is better able to return a serve having just observed a serve on the same side.
- The additive bonus/penalty to the server depends on whether the receiver has prepared for this strategy or not (by leaning in that direction).
- To match this sports example and interpret payoffs as the chance of winning a point, assume \(0 \leq y < x < 1\). To avoid state contingent dominant strategies, assume \(\varepsilon\) and \(\xi\) are relatively small, i.e. \(\max\{|\varepsilon|, |\xi|\} < \min\{x - y, 1 - x\}\).
- The realized actions of player 1 is commonly observable.
- All of the above is common knowledge.

\[
\begin{array}{c|cc}
\theta = L & \ell & r \\
L & y + \varepsilon & x + \xi \\
R & x & y \\
\end{array} \quad \begin{array}{c|cc}
\theta = R & \ell & r \\
L & y & x \\
R & x + \xi & y + \varepsilon \\
\end{array}
\]

Figure 1: State Contingent Stage Game Payoffs. Stage game payoffs are given by the left matrix following a \(L\) serve, and by the right matrix following a \(R\) serve. The listed payoffs are for the server (row), the receiver (column) receives the minus of these payoffs.
2 The Unique Markov Equilibrium

- Given common knowledge of the state, the equilibrium will involve state contingent mixture chances in each state.

- Let $\beta_\theta$ be the weight the serve receiver places on $\ell$ in state $\theta$. In equilibrium:
  \[
  \beta_L = \frac{x - y + \xi}{\xi - \varepsilon + 2(x - y)} \quad \text{and} \quad \beta_R = \frac{x - y - \varepsilon}{\xi - \varepsilon + 2(x - y)}
  \]

- Let $\alpha_\theta$ be the weight the server places on $L$ in state $\theta$. In equilibrium:
  \[
  \alpha_L = \frac{x - y}{\xi - \varepsilon + 2(x - y)} \quad \text{and} \quad \alpha_R = \frac{x - y + \xi - \varepsilon}{\xi - \varepsilon + 2(x - y)}
  \]

- These mixtures make each player myopically indifferent. One might think this cannot be an equilibrium, for surely the server prefers one of the states over the other. But in this symmetric world this is not so as the one-shot state contingent value to the server is equalized across states:
  \[
  V_L = V_R = \frac{x(x + \xi) - y(\varepsilon + y)}{\xi - \varepsilon + 2(x - y)}
  \]

  [Symmetry simplifies the analysis substantially. In a richer model the server would trade off expected gains/losses on the current point for a less/more advantageous stage game on the next point.]

- The probability the receiver plays $\ell$ in period $t + 1$ does not depend on the strategy the receiver plays in period $t$.

- The server’s strategy is negatively serially correlated provided:
  \[
  \alpha_R > \alpha_L \iff \xi > \varepsilon
  \]

  and positively serially correlated provided $\alpha_R < \alpha_L \iff \xi < \varepsilon$.  
