Opportunistic Matching in the Housing Market*

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1 Introduction

In many markets, it takes buyers and sellers time to find one another. Once contact occurs, whether a transaction is consummated and if so at what price depends on the circumstances of the potential traders. In the housing market, for example, the willingness of a prospective buyer to pay a given price for a particular house depends on the buyer's income as well as her current housing arrangements, re-location deadlines, the likelihood of finding more desirable properties, the availability of alternative rental accommodation, and so on. Sellers have similar concerns. Such differences in the *flow values* that buyers and sellers receive while unmatched are an important – perhaps the most important – reason that some sales are made while others are not. Moreover, they are likely to affect the terms of trade.

Circumstances not only vary across buyers and sellers, they can also change over time, often for the worse. A buyer can initially afford to be selective, but eventually, if she is unable to purchase a house on favorable terms, her search becomes more desperate. Likewise, a typical seller entering the housing market can at first afford to hold out for a high price but eventually, if he cannot find a buyer willing to pay his price, he becomes what real estate ads often call "a motivated seller." That is, both buyers and sellers experience undesirable transitions if they do not buy or sell quickly enough.

To assess the effects of such transitions on the housing market, we build a search/matching

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model in which both buyers and sellers experience, if they stay in the market long enough, a decline in the flow value of continuing to search. The emphasis on such transitions distinguishes our model from earlier search-theoretic work on the housing market.[1] Moreover, these transitions, as we argue below, not only frequently occur in this market, but their incorporation into our model yields implications – especially those regarding the distribution of prices conditional on time to sale – that correspond to data commonly observed for housing markets.

The model is straightforward. Agents enter the market in a relaxed state with a high flow value from being unmatched. Over time, an agent who does not buy or sell, eventually falls into a desperate state with a flow value that is normalized to zero. These transitions in the flow value of continued search, which occur at an exogenous Poisson rate, generate heterogeneity among buyers and sellers. As time proceeds, prospective buyers and sellers randomly meet one another and decide whether to trade. When a house is sold, the buyer pays the seller a price – determined through Nash bargaining – which depends on the buyer and seller types. The value of the house is independent of the two agents' types.

Not all meetings, however, necessarily lead to transactions. A match occurs, i.e., a house moves from seller to buyer, if and only if the sale results in an increase in the sum of the values for the two parties; otherwise, the potential buyer and seller continue to search. The continuation flow values that the prospective buyer and seller bring to the table determine whether this surplus exists and therefore whether an exchange takes place. When a desperate seller, i.e., one with a low flow value, meets a prospective buyer, she is willing to accept a relatively low price and so is more likely to sell. Similarly, when a desperate buyer finds a prospective seller, he is more likely to buy. Thus, the outcome of a meeting between a prospective buyer-seller pair, i.e., whether a sale occurs and, if so, at what price, depends on the buyer's and seller's types, which in turn are related to their durations of search.[2]

A rich set of implications arise. Suppose first that buyers and sellers have symmetric flow values when unmatched. We show in this symmetric case that only three equilibrium matching patterns can occur. If the value of the house is sufficiently large, indiscriminate

[1] See, for example, Wheaton (1990), Williams (1995), or Krainer and LeRoy (2002).

[2] In order that not all encounters lead to transactions, some form of heterogeneity must exist. Heterogeneity may be match-specific as in Williams (1995). Another possibility is that individuals are inherently different, i.e., there is a distribution of types across new entrants. This approach is commonly taken in the search/matching literature, where it is usually combined with the assumption of complementarity in producing the match output. These models then focus on heterogeneity in match values. This is the approach taken in Becker (1973), Burdett and Coles (1997), and Shimer and Smith (2000). Our approach is to generate a distribution of types in flow values resulting from experience while unmatched. The output produced by the match (the value of a house) however is not heterogeneous.

matching (IM) in which every encounter leads to a sale obtains. For lower house values, there is an equilibrium, which we term opportunistic matching (OM), in which relaxed searchers wait for desperate partners, while the desperate match with anyone. If the house value is sufficiently small, an equilibrium in which only desperate agents match (DM) arises. Finally, for some house values, multiple equilibria (either OM or DM) are possible. In the general (asymmetric) case, when the flow value of relaxed buyers and the flow value of relaxed sellers are not the same, two additional equilibrium matching patterns are possible. In no case does the model generate either strictly positive assortative matching (relaxed only match with relaxed and desperate only match with desperate) or strictly negative assortative matching (relaxed only match with desperate and vice versa).

Exploring the model's implications in more detail, we derive the marginal distribution of price, the marginal distribution of time to sale, and the conditional distribution of price given time to sale for the three symmetric equilibrium types (IM, OM, and DM). We analyze how expected price and price dispersion, both unconditionally and conditional on time to sale, vary with model parameters, both within and across equilibrium types. Similarly, we show how expected time to sale depends on model parameters.

In constructing our model, we have made choices about which features of the housing market to emphasize and which to abstract from. We chose to make agents heterogeneous in terms of search values; we could have chosen heterogeneity in transaction values. We chose a complete information framework and determined prices with Nash Bargaining; we could have made agent types private information and made specific assumptions about price negotiation.[3] While we feel our model highlights an important, previously unmodeled feature of the housing market and has a rich set of testable implications, we do not mean to suggest that the alternative modeling choices are not appealing. In particular, private information is clearly an important and interesting feature of the housing market. We assume complete information in order to highlight the effects of transitions in flow values without the added difficulties that private information would entail. However, by assuming complete information, we rule out *a priori* any implications for the details of price negotiations between buyers and sellers since the Nash Bargaining solution obscures such details in its black box. As a result, we surely miss some important features of sales price negotiations.[4] (See Merlo and Ortalo-Magné 2004

[3]For example, Krainer and LeRoy (2002) assume that sellers make take-it-or-leave offers in a private information setting. Modeling price negotiation in this way has some limitations. In particular, it implies that the sales price depends only on the seller's type.

[4]Arnold (1999) explicitly models price negotiations. His model, which unlike ours is not a market model, considers a single seller who posts an asking price that influences the arrival rate of potential buyers. A buyer's valuation is realized upon arrival and becomes common knowledge. If there are potential gains from trade,

for a detailed study of housing transactions data.)

Our assumption of a Poisson transition from relaxed to desperate is meant to capture the various factors that alter reservation values of search, e.g., learning, borrowing constraints, etc. while at the same time giving tractable analytic results. An alternative would be to explicitly model the dynamics of individuals' reservation values. In the labor search literature, this has been done by van den Berg (1990), which incorporates the limited duration of unemployment benefits, and by Burdett and Vishwanath (1988), which allows for learning about the distribution of wage offers, among others. Neither of these papers incorporates this individual decision-theoretic analysis into an equilibrium model because the resulting model would be intractable. The assumption of a Poisson transition is a commonly used modeling device that avoids complicated state dependence while still allowing a correlation between time on the market and market outcomes. This assumption allows us to generate interesting equilibrium results. For example, our results on the distribution of prices and on multiple equilibria would simply not be possible absent the Poisson assumption.

The real test of simplifying assumptions is how useful the model is in explaining the observed market outcomes. The foremost stylized fact from the housing market transaction data is that, conditional on house characteristics, sales price declines with time on the market. Merlo and Ortalo-Magné (2004, p. 214) write "*Ceteris paribus*, the longer the time on the market, the lower the sale price. This is a well known empirical finding ...."[5] This result arises in our IM and OM equilibria. Merlo and Ortalo-Magné (2004) also present data on the housing sales process, i.e., offers made, accepted, and rejected. They find that there are unconsummated matches; specifically, they find that 1/3 of all negotiations do not lead to a sale. This is consistent with OM and DM equilibria if one assumes that the failure to reach a deal is the result of a lack of joint surplus resulting from the sale rather than the result of inefficiencies associated with private valuations. They also find that the vast majority of sellers who fail to reach agreement in their first negotiation ultimately receive a higher price, but a significant fraction end up accepting a lower price. This is consistent with the behavior of a relaxed seller who chooses not to match with a relaxed buyer. The seller may get a better price later from a more desperate buyer. If the seller changes state before selling the house, however, and becomes a desperate seller, the house will sell at a lower price. Finally, Merlo and Ortalo-Magné (2004) find that about 1/4 of all sellers make infrequent (typically one) but

bargaining then proceeds with the asking price as the seller's initial offer. Like our model, Arnold (1999) cannot capture the fact that negotiations, once started, sometimes fail to lead to agreement.

[5]This is consistent with earlier studies, such as Miller and Sklarz (1987), that show that as time on the market increases, the spread between the list price and sale price increases. Horowitz (1992) shows that list price is a good predictor of sale price.

sizable changes in the list price during the time the house is on the market. These changes are almost always price decreases and are usually substantial, which supports the notion of a transition from relaxed to desperate. Similarly, Glower et al (1998) survey home sellers and find that sellers are heterogeneous in their motivations. They argue that the empirical evidence calls for a model in which some sellers are motivated to sell quickly, while others are not. This also supports our modeling these transitions as changes in motivation.[6]

We present the details of the model in Section 2. Then, in Section 3, we fully characterize the equilibrium correspondence. That is, we establish when each of five possible equilibrium matching configurations occurs. In Section 4 we investigate the joint distribution of prices and time to sale. Finally, in Section 5, we conclude and suggest several ways in which our relatively simple framework could be extended to add additional realism.

2 Model

Equal numbers of buyers and sellers enter the market at an exogenous rate. They enter in the relaxed state with a high flow value from being unmatched. Over time, if an agent does not buy or sell, he or she eventually moves from the relaxed into the desperate state at an exogenous Poisson rate λ. The flow value for a relaxed buyer is $\beta > 0$; the corresponding value for a desperate buyer is normalized to zero. Similarly, sellers have flow values $\sigma > 0$ and zero. Let ϕ_b denote the fraction of buyers who are desperate; let ϕ_s be the fraction of sellers who are desperate.

Prospective traders meet each other at the exogenous constant Poisson rate α. If the match value of the house, x, exceeds the sum of their opportunity costs – their values of continued search – a trade takes place at a price determined by symmetric Nash bargaining. As the payoff to continued search depends on the buyer's and seller's states, so too does the price. If a type b buyer and a type s seller agree to a sale, the seller receives price $p(b,s)$, while the buyer realizes value $x - p(b,s)$. The pair then exits the market.

The value for a relaxed buyer satisfies

$$\begin{aligned} rB^R &= \beta + \alpha\phi_s \max\{0, x - p(R,D) - B^R\} \\ &\quad + \alpha(1-\phi_s)\max\{0, x - p(R,R) - B^R\} + \lambda(B^D - B^R). \end{aligned}$$

She receives flow value β while unmatched, and meets a prospective seller at rate α. That

[6] Zuehlke's (1987) finding that vacant houses are more likely to sell the longer they are on the market, while occupied houses do not exhibit duration dependence is also consistent with our relaxed/desperate dichotomy.

seller is desperate with probability ϕ_s, and a match is formed if and only if the joint surplus is positive. In that case, the buyer receives value $x - p(R, D)$ and gives up value B^R. Otherwise, she continues unmatched. With probability $1 - \phi_s$, the prospective seller she meets is relaxed, a match is formed if and only if the joint surplus is positive; otherwise, she continues to search. Finally, she moves to the desperate state at rate λ and receives the value B^D, defined by

$$rB^D = \alpha\phi_s \max\{0, x - p(D, D) - B^D\} + \alpha(1 - \phi_s) \max\{0, x - p(D, R) - B^D\}.$$

On the sellers' side, the values are

$$\begin{aligned} rS^R &= \sigma + \alpha\phi_b \max\{0, p(D, R) - S^R\} \\ &\quad + \alpha(1 - \phi_b) \max\{0, p(R, R) - S^R\} + \lambda(S^D - S^R) \\ rS^D &= \alpha\phi_b \max\{0, p(D, D) - S^D\} + \alpha(1 - \phi_b) \max\{0, p(R, D) - S^D\}. \end{aligned}$$

It follows from these value equations that $B^R > B^D$ and $S^R > S^D$. Since the relaxed type enjoys a positive flow value from being unmatched, while the desperate type does not, the relaxed type must have a strictly higher expected present discounted value.

Finally, prices are determined by symmetric Nash bargaining, i.e.,

$$p(b, s) = \frac{1}{2}(x - B^b + S^s).$$

A steady-state equilibrium is a matching pattern such that (i) pairs of agents form matches if and only if the joint surplus from doing so is positive and (ii) the appropriate steady-state conditions are satisfied.[7] We consider only pure-strategy equilibria. There are several potential equilibrium types to consider. Since the prospective buyer and seller can each be either relaxed or desperate, four possible pairings of types emerge. Since each of these potential pairings either matches or not, there are sixteen (2^4) possible matching patterns.

Many of these potential matching patterns can be eliminated *a priori* as possible steady-state equilibrium outcomes, leaving the five possible equilibrium configurations depicted in Figure 1. As discussed in the introduction, three of these five equilibria – IM, OM, DM – are symmetric and are the only equilibria that can occur when buyers and sellers have symmetric unmatched flow values. If the unmatched flow values are asymmetric, two other equilibria are possible. Under buyer opportunistic matching (BOM), all buyers match only with desperate

[7] Existence of steady-state equilibria in models such as ours involves technical difficulties solved in Duffie and Sun (2004).

		IM		OM		BOM		SOM		DM	
		R	D	R	D	R	D	R	D	R	D
	R	●	●	○	●	○	●	○	○	○	○
Buyer	D	●	●	●	●	○	●	●	●	○	●

Figure 1: Possible Equilibrium Matching Patterns. The buyer types are in rows, while the seller types are in columns. A filled in circle indicates that matches between the relevant types of buyers and sellers form in the given equilibrium.

sellers, while relaxed sellers do not match with anyone. BOM obtains in equilibrium for middling values of x, when the flow value for relaxed sellers is relatively high. Symmetrically, seller opportunistic matching (SOM) obtains when sellers only match with desperate buyers.

Lemma 1 *There are only five possible equilibrium configurations. These five matching patterns are depicted in Figure 1.*

To establish Lemma 1, note that $B^R > B^D$ and $S^R > S^D$ imply that whenever a relaxed type matches in equilibrium, his or her desperate counterpart must as well. For example, a relaxed seller matches with a type θ buyer if $S^R + B^\theta \leq x$. If $S^R > S^D$, then $S^D + B^\theta < x$ must also hold; i.e., the desperate seller must also match with the type θ buyer. Second, note that $x > 0$ implies at least one desperate type must match in equilibrium, else $S^D = B^D = 0$, and desperate types could strictly improve their payoffs by matching with each other. These insights rule out any equilibrium matching pattern not displayed in Figure 1.

We emphasize that Lemma 1 and Figure 1 rule out *strict positive assortative matching (PAM)* and *strict negative assortative matching (NAM)*. PAM obtains when only like types match, i.e., relaxed agents only match with relaxed agents and desperate agents only match with desperate agents. NAM obtains when agents only match with agents of the opposite type. These two potential equilibrium types often arise in models with heterogeneity in match values. In models such as Becker (1973) with heterogeneous match values, no frictions, and transferable utility, when output is supermodular in types, i.e., types are complements in production, PAM must obtain. With search frictions, PAM follows from assuming sufficient complementarity (Shimer and Smith 2000). If utility is not transferable, then in a frictionless matching model, PAM follows from monotonicity rather than complementarity assumptions. That is, if each type's return to matching rises in the type of their partner, then PAM obtains. In our model, there is no heterogeneity in match values - every match yields the same value, x. The impossibility of PAM and NAM follows from a simple dominance argument. Since match values are the same and relaxed types have higher flow values, relaxed types must have higher unmatched values than desperate types. As a result, whenever (R,R) matches form, (R,D) matches must form as well, blocking PAM. Similarly, whenever (R,D) matches form, (D,D) matches must also form, blocking NAM.

For the remaining five potential equilibrium types, we need to consider the steady-state conditions. These are simple: the flow into the desperate state must equal the flow out of the desperate state (to the matched state). Let ϕ_s and ϕ_b be the steady-state fraction of sellers and buyers who are desperate. The rate at which agents flow into the desperate state is $\lambda(1-\phi_k)$ for $k \in \{b, s\}$ regardless of the matching pattern. However, the rate at which agents flow out of the desperate state depends on the matching pattern.

In the IM and OM cases, every meeting for a desperate agent results in a match, and the outflow rate is $\alpha\phi_k$. Setting $\lambda(1-\phi_k) = \alpha\phi_k$, we find

$$\phi \equiv \phi_s = \phi_b = \frac{\lambda}{\alpha + \lambda}.$$

In the DM case, we still have $\phi_s = \phi_b$, but not equal to ϕ. Let ϕ^D be the fraction of desperate agents in this case. The outflow is $\alpha(\phi^D)^2$, since a meeting only results in a sale if both agents are desperate. Setting $\lambda(1-\phi^D) = \alpha(\phi^D)^2$, we determine

$$\phi^D = \frac{-\lambda + \sqrt{\lambda^2 + 4\alpha\lambda}}{2\alpha}.$$

Note that $\phi^D > \phi$. Intuitively, when it is harder to exit the desperate state, there is a larger fraction of desperate agents in steady state. We leave the asymmetric cases (SOM and BOM) for the Appendix, to avoid cluttering the text.

3 Equilibrium Matching Patterns

In this section we characterize the equilibrium matching correspondence, i.e. we show which matching patterns are sustainable for different values of the exogenous parameters.

3.1 Symmetric Case Summary

For simplicity, normalize $\beta = \sigma = 1$. Proposition 1 summarizes the symmetric case.

Proposition 1 *In the symmetric case, there are three equilibrium matching patterns. The correspondence between x and the equilibrium matching patterns is shown in Figure 2.*

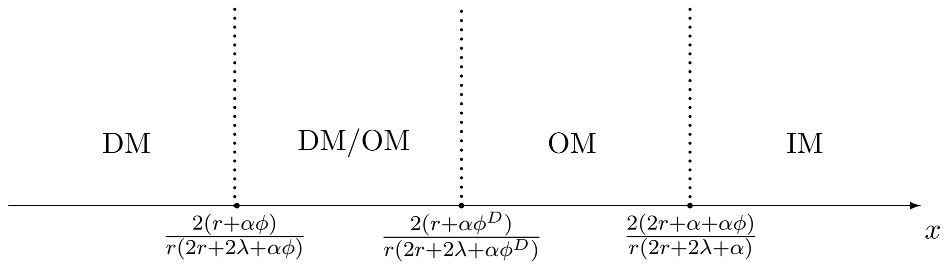

Figure 2: Symmetric Equilibrium Correspondence. We have graphed the set of equilibria for differing values of x, assuming $\beta = \sigma = 1$.

Figure 2 gives the possible equilibrium matching patterns for different values of x. For now, ignore the specific threshold values of x, and instead focus on the general pattern. Not surprisingly, if x is high enough, all meetings result in a match (IM) while if x is low enough, then the converse holds: no relaxed agents match, and DM obtains.

Intermediate values of x are more interesting. For these intermediate values, OM is an equilibrium. Although the match value x is not high enough to induce relaxed agents to match with one another, it is high enough so that desperate agents are willing to compensate relaxed agents to make it worth their while to match. At the lower end of the intermediate values of x, there are multiple equilibria (OM or DM). This results from a compositional effect. In DM equilibrium, the steady-state fraction of desperate agents is higher than in OM equilibrium ($\phi^D > \phi$). The intuition for why this causes multiplicity is as follows. All agents are better off meeting a desperate agent rather than a relaxed one. Consider a potential match between a desperate buyer and a relaxed seller. This match is consummated if and only if $x \geq B^D + S^R$. If the DM equilibrium obtains, all other (R, D) and (D, R) matches fail to form, so there are more desperate agents in the market and $B^D + S^R$ is high relative to the case in which these matches do form. Thus, DM is a viable equilibrium pattern. If, on the other hand, all other (R, D) and (D, R) matches form, then $B^D + S^R$ becomes low relative to the case in which these matches do not form and the now fewer desperate searchers are more willing to trade. OM is then also a viable equilibrium pattern. Thus, the individual willingness of desperate buyers or sellers to trade with relaxed agents generates a spillover that reinforces their decisions and causes a coordination problem. [8]

[8] Note that in addition to the compositional effect, there is another effect of having (R, D) and (D, R) matches form. Holding the fraction of desperate agents constant, the fact that these matches form raises the values of being unmatched. However, at the upper threshold, $x = B^D + S^R$ in DM equilibrium, so there is no net surplus to these matches and this effect is zero. Below the threshold (in the region of multiplicity), the compositional effect then dominates.

3.2 Symmetric Case Details

For each of the three cases, we solve for the equilibrium value functions assuming existence and then determine the parameter configurations that are consistent with the given equilibrium.[9]

Indiscriminate Matching. For all matches to be consummated, we must have $x - B^b - S^s \geq 0$ for all combinations of (b, s). Assuming this is the case and substituting the symmetric Nash bargaining expressions for prices, we have the following values:

$$\begin{aligned} B^R &= S^R = \frac{4r + \alpha(\alpha x + 2\phi + 2\lambda x + 2rx + 2)}{2\,(2r + \alpha + 2\lambda)\,(r + \alpha)} \\ B^D &= S^D = \frac{\alpha(\alpha x + 2\phi + 2\lambda x + 2rx - 2)}{2\,(2r + \alpha + 2\lambda)\,(r + \alpha)}. \end{aligned}$$

Notice that $B^R = S^R$ and $B^D = S^D$ follow from assuming symmetric Nash bargaining.

We now need to determine the parameter configurations that are consistent with an IM equilibrium. Since B^R $(= S^R) > B^D$ $(= S^D)$, this reduces to checking when $x - 2B^R \geq 0$. Substituting our derived value for B^R, IM is an equilibrium if and only if

$$x \geq \frac{2(2r + \alpha + \alpha\phi)}{r(2r + 2\lambda + \alpha)}.$$

This is the threshold value between IM and OM given in Figure 2.

Opportunistic Matching. We determine the conditions for an OM equilibrium in a similar fashion. First, we assume that OM is an equilibrium and then substitute the Nash bargaining solution for prices to give

$$\begin{aligned} B^R &= S^R = \frac{4r + \alpha(2r\phi x + 2\phi + 2\lambda x + 2 + \alpha\phi^2 x)}{2(2r^2 + 2r\lambda + r\alpha + 2\alpha\phi r + 2\alpha\lambda + \alpha^2\phi^2)} \\ B^D &= S^D = \frac{\alpha\left(2xr + 2\phi + 2\lambda x - 2 + \alpha\phi^2 x\right)}{2(2r^2 + 2r\lambda + r\alpha + 2\alpha\phi r + 2\alpha\lambda + \alpha^2\phi^2)}. \end{aligned}$$

Then we check what conditions ensure $x \leq B^R + S^R$, while at the same time, $x \geq B^D + S^R$ $(= B^R + S^D)$.

(i) For $x \leq B^R + S^R$, we need $x \leq \dfrac{2(2r + \alpha + \alpha\phi)}{r(2r + 2\lambda + \alpha)}$, and

(ii) for $x \geq B^D + S^R$, we need $x \geq \dfrac{2(r + \alpha\phi)}{r(2r + 2\lambda + \alpha\phi)}$.

[9] In an earlier version of the paper, we showed that there is not enough opportunistic matching in equilibrium to maximize the present discounted value of new entrants in steady state.

Straightforward algebra establishes that the interval for x given by (i) and (ii) is nonempty. The condition on x given in (i) is precisely the opposite of the condition required for an IM equilibrium. These are the thresholds that we have graphed in Figure 2.

Desperate Matching. Assuming that only desperate agents match[10] and substituting symmetric Nash bargaining expressions for prices, one can solve for the following values:

$$\begin{aligned} B^R &= S^R = \frac{2(r+\alpha\phi^D) + \alpha\phi^D x\lambda}{2\left(r+\alpha\phi^D\right)(r+\lambda)} \\ B^D &= S^D = \frac{\alpha\phi^D x}{2\left(r+\alpha\phi^D\right)}. \end{aligned}$$

Since $B^R = S^R$ and $B^D = S^D$, the necessary conditions for DM equilibrium are

$$x - 2B^R \leq 0, \qquad x - B^R - B^D \leq 0, \qquad x - 2B^D \geq 0.$$

Since $B^R > B^D$, we only need to show that $x - B^R - B^D \leq 0$ and $x - 2B^D \geq 0$.

(iii) For $x \leq B^R + B^D$, we need $x \leq \dfrac{2(r+\alpha\phi^D)}{r(2r+2\lambda+\alpha\phi^D)}$, and

(iv) since $2B^D = \dfrac{\alpha\phi^D x}{(r+\alpha\phi^D)}$, it is obvious that $x \geq 2B^D$.

Condition (iii) is the same as the lower threshold in the OM case, except that we have replaced ϕ with ϕ^D. Since $\phi^D > \phi$, the thresholds are ordered as given in Figure 2. This reflects the compositional effect noted above.

3.3 Asymmetric Case Summary

Now consider the asymmetric case with $\beta \neq \sigma$. We summarize the results here and work out the algebraic details in the Appendix. Some of the features of the symmetric case extend to the more general case. Equilibrium always exists. For some values of the parameters, equilibrium is unique, while for others there are two equilibria. There are never more than two equilibria. As before, IM is the only equilibrium for high x, while DM is the only equilibrium for low x.

For intermediate values of x, the situation is more complex. Figure 3 presents the set of possible equilibria for different values of (rx, β, σ) normalized such that $rx + \beta + \sigma = 1$.

[10] Although they never match in DM equilibrium, relaxed agents are assumed to search. This is a strong assumption, but it keeps the model simple. A slight perturbation of the model would give the relaxed cause to search. For example, if buyers received a match-specific value in addition to x, then, with $\beta > 0$, they would search for the "perfect" house even if almost all matches involved desperate agents. Similarly, relaxed sellers would search for the "perfect" buyer.

This normalization allows us to use an equilateral triangle with height 1 to represent the equilibrium correspondence. Any triple (rx, β, σ) is represented as a point in this triangle such that each variable is equal to the minimum distance between the point and the side of the triangle opposite the variable's label. For example, the base of the triangle contains all combinations for which $\sigma = 0$, with x increasing moving right to left along the base.

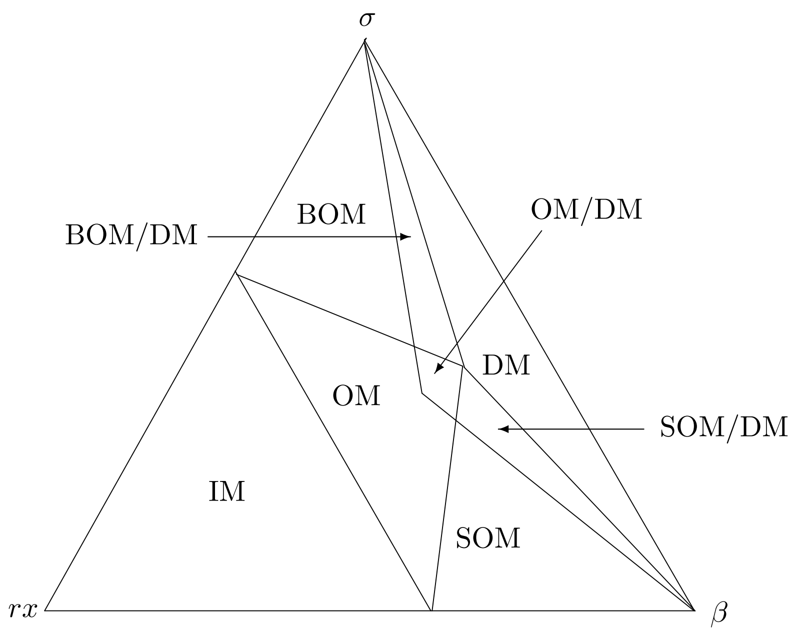

Figure 3: Equilibrium Correspondence. We have graphed the set of equilibria for different values of (rx, β, σ), setting $\beta + \sigma + rx = 1$.

To recover the symmetric case, draw the $\beta = \sigma$ line in the diagram — this line is the x-axis in the symmetric case. Note that this line does not cross through the SOM or BOM regions of the diagram. When IM is an equilibrium, it is unique, while the DM and OM ranges continue to overlap when β and σ are not too far apart. The SOM and BOM regions do not overlap with each other (or with the OM region), but each overlaps with the DM region.

Proposition 2 *In general, there are five equilibrium matching patterns. The correspondence between x, β, and σ and the equilibrium matching patterns is shown in Figure 3.*

4 The Distributions of Price and Time to Sale

Price and time to sale are the key observables in housing market data. In this section, we investigate the joint distribution of these two variables in the symmetric case. We do this by first deriving the marginal distributions of price and of time to sale. Then we find the conditional distribution of price given time to sale.

4.1 The Price Distribution

The desperate matching case is trivial. There is only one price at which transactions occur, $p(D, D) = x/2$. We consider the remaining cases below.

Indiscriminate Matching. We substitute the solution determined above into the Nash bargaining conditions to find

$$\begin{aligned} p(D,D) &= p(R,R) = \frac{x}{2} \\ p(R,D) &= \frac{x}{2} - \frac{1}{2r+\alpha+2\lambda} \\ p(D,R) &= \frac{x}{2} + \frac{1}{2r+\alpha+2\lambda}. \end{aligned}$$

The fraction of transactions that take place at $x/2$ is $\phi^2 + (1-\phi)^2$. The fraction that take place at the higher price, $p(D, R)$, is $(1-\phi)\phi$, as is the fraction that take place at the lower price, $p(R, D)$. The mean (and median) price is $x/2$.

The comparative statics are straightforward. All prices rise with x. Increases in the other parameters, α, λ, and r, all cause the high price to fall and the low price to rise. Changes in α and λ also change the fractions of transactions at the various prices. Recall that $\phi = \frac{\lambda}{\alpha+\lambda}$ so $\phi_\alpha < 0$ and $\phi_\lambda > 0$. The fraction of transactions at the extreme prices, i.e., at $p(R, D)$ and $p(D, R)$, equals $2(1-\phi)\phi$. Thus, if $\lambda < \alpha$, i.e., if $\phi < \frac{1}{2}$, an increase in α unambiguously reduces price dispersion, while an increase in λ has countervailing effects. When $\lambda > \alpha$, the comparative statics are reversed.

Opportunistic Matching. Substitution yields

$$\begin{aligned}
p(D,D) &= \frac{x}{2} \\
p(R,D) &= \frac{x}{2} - \frac{2(r+\alpha) - r\alpha x(1-\phi)}{2(2r^2 + 2r\lambda + r\alpha + 2\alpha\phi r + 2\alpha\lambda + \alpha^2\phi^2)} \\
p(D,R) &= \frac{x}{2} + \frac{2(r+\alpha) - r\alpha x(1-\phi)}{2(2r^2 + 2r\lambda + r\alpha + 2\alpha\phi r + 2\alpha\lambda + \alpha^2\phi^2)}.
\end{aligned}$$

The mean price is again $x/2$. The fraction of contacts that lead to sales is $1-(1-\phi)^2$ since (R,R) matches do not form. The fraction of sales that take place at $p(D,D)$ is thus $\frac{\phi^2}{1-(1-\phi)^2}$, the fraction at $p(R,D)$ is $\frac{(1-\phi)\phi}{1-(1-\phi)^2}$, and the fraction at $p(D,R)$ is also $\frac{(1-\phi)\phi}{1-(1-\phi)^2}$. In this case, an increase in ϕ unambiguously increases the fraction of prices at the mean; i.e., an increase in α leads to an increase in the fraction of prices in the tails while an increase in λ leads to the reverse.

The comparative statics for the prices are different than in the IM case. First, an increase in x not only increases the mean price, but it also decreases price dispersion. An increase in r leaves the mean price unchanged but decreases the range of prices.[11] As noted above, α and λ affect the distribution of prices via their effect on the proportions of prices in each category. An increase in λ decreases the range of prices and, as we noted above, it also decreases dispersion by reducing the weights in the tails of the distribution. Changes in α have an ambiguous effect on the range of prices.[12]

Comparisons Across Equilibria. Comparing marginal price distributions across equilibria, holding all exogenous parameters constant, does not make sense. For example, if a configuration of parameters implies that IM is the unique equilibrium, then substituting the parameters into the formulae for the equilibrium prices in the OM case is nonsensical. Instead we look at how the price distribution changes as we change exogenous parameters, taking into account changes in the equilibrium matching structure. In particular, we focus on the effect of varying x.

In all equilibria, the mean price (the only price in the DM equilibrium) is the same, and the other two prices (in the IM and OM equilibria) are equidistant from the mean. The probability that a low or high price is realized is not a function of x in any case. Thus, we

[11] To see this, evaluate $\partial(p(D,D)-p(R,D))/\partial r$ at the lower and upper boundary of the OM range (in x). In both cases the result is negative. Then note that $\partial(p(D,D)-p(R,D))/\partial r$ is monotonic in x.

[12] It turns out that $\partial(p(D,D)-p(R,D))/\partial\alpha$ is negative at the lower boundary of the OM range. At the upper boundary it is positive for high α and low r.

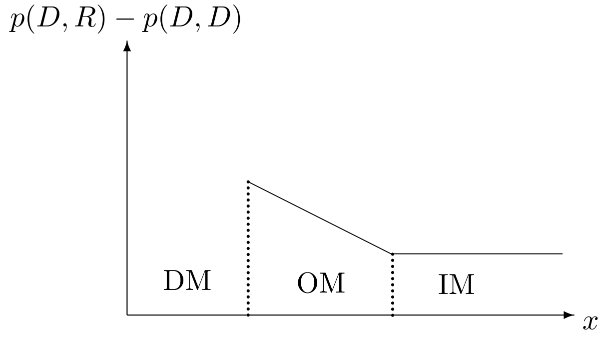

Figure 4: Price Spread. The spread in prices as a function of x, assuming OM (vs. DM) obtains throughout its allowable range.

can fully describe the change in the distribution of prices as a function of x by investigating what happens to the spread in prices, $p(D,R)-p(D,D)$. For low values of x, DM obtains, and this spread is zero. Under OM, the spread falls linearly in x. Over the IM region, the spread is constant in x. Finally, for x at the boundary between the OM and IM regions, the price spread is the same regardless of which equilibrium (OM or IM) occurs. See Figure 4 for a summary.

4.2 Time to Sale

In this section, we calculate the expected time to sale for each of the three symmetric equilibria. If $G(t)$ equals the probability that no sale occurs by time t, then $\int_0^\infty G(t)dt$ equals the expected time to sale. Conditional on the transition between relaxed and desperate occurring at time τ, we have

$$G(t|\tau) = \begin{array}{ll} e^{-\alpha\mu_R t} & \forall\ t \le \tau \\ e^{-\alpha\mu_R \tau}e^{-\alpha\mu_D(t-\tau)} & \forall\ t > \tau \end{array},$$

where $G(t|\tau)$ is the probability that no sale occurs by time t conditional on the transition to desperation occurring at time τ, and μ_R and μ_D are the fractions of meetings that lead to a sale for relaxed and desperate sellers, respectively.

The density over transition times τ is $\lambda e^{-\lambda\tau}$. So we can compute

$$G(t) = \int_0^\infty G(t|\tau)\lambda e^{-\lambda\tau}d\tau = \frac{\alpha(\mu_R-\mu_D)e^{-(\alpha\mu_R+\lambda)t}+\lambda e^{-\alpha\mu_D t}}{\alpha(\mu_R-\mu_D)+\lambda}.$$

(The density over time to sale can be found by differentiating the negative of $G(t)$.) Finally, we integrate $G(t)$ over t to determine the expected time to sale

$$E[T] = \int_0^\infty G(t)dt = \frac{\alpha\mu_D + \lambda}{\alpha(\alpha\mu_D\mu_R + \lambda\mu_D)}.$$

We then substitute for μ_R and μ_D to determine the expected durations in each of the three symmetric equilibria, as summarized in Table 1. One can show that the equilibria are ordered by expected time to sale from longest to shortest: desperate matching, opportunistic matching, and indiscriminate matching.

Matching	μ_R	μ_D	$E[T]$
Indiscriminate	1	1	$1/\alpha$
Opportunistic	ϕ	1	$[(\alpha+\lambda)^2]/[\alpha\lambda(2\alpha+\lambda)]$
Desperate	0	ϕ_d	$1/\lambda + 2/[\sqrt{\lambda(4\alpha+\lambda)} - \lambda]$

Table 1: Expected Time to Sale. This table gives the expected time to sale in each of the three cases.

4.3 The Distribution of Price Given Time to Sale

We are interested in the distribution of price conditional on time to sale since this is what one typically observes in data. In DM equilibrium, the price is $p(D, D) = x/2$ with probability one. The IM and OM cases are more interesting, and we consider them in turn.

Indiscriminate Matching. Consider a transaction by a vintage t seller, i.e., a seller whose completed duration is t. In order to compute the conditional distribution of price given t, we need to know the probability that the seller was desperate conditional on t: $Pr(s = D|t)$. With indiscriminate matching, each type of seller is equally likely to transact, thus $Pr(s = D|t) = \phi(t)$, where $\phi(t)$ is the proportion of vintage t sellers who are desperate. The chance of remaining on the market to time t and remaining relaxed is thus $e^{-(\alpha+\lambda)t}$, while the chance of remaining on the market and being desperate is $(1 - e^{-\lambda t})e^{-\alpha t}$, so

$$\phi(t) = 1 - e^{-\lambda t}.$$

Recall that the proportion (across all vintages) of desperate buyers in the market is $\phi = \lambda/(\alpha+\lambda)$, so the conditional distribution of price given t is

$$\begin{aligned}
f(p(R,D)|t) &= (1-\phi)\phi(t) = \frac{\alpha(1-e^{-\lambda t})}{\alpha+\lambda} \\
f(p(D,D)|t) &= \phi\phi(t) = \frac{\lambda(1-e^{-\lambda t})}{\alpha+\lambda} \\
f(p(R,R)|t) &= (1-\phi)(1-\phi(t)) = \frac{\alpha e^{-\lambda t}}{\alpha+\lambda} \\
f(p(D,R)|t) &= \phi(1-\phi(t)) = \frac{\lambda e^{-\lambda t}}{\alpha+\lambda}.
\end{aligned}$$

Since $p(D,D) = p(R,R) = x/2$, the fraction of transactions occurring at price $x/2$ equals $f(p(D,D)|t) + f(p(R,R)|t) = \dfrac{\lambda + (\alpha-\lambda)e^{-\lambda t}}{\alpha+\lambda}$.

Opportunistic Matching. This case is more complicated since relaxed sellers and desperate sellers transact at different rates, which complicates the calculation of the probability that the seller was desperate conditional on transacting at vintage t. Bayes rule yields

$$\begin{aligned}
Pr(s=D|t) &= \frac{\Pr(t|s=D)\Pr(s=D)}{\Pr(t|s=D)Pr(s=D)+\Pr(t|s=R)Pr(s=R)} \\
&= \frac{\alpha\phi(t)}{\alpha\phi(t)+\alpha\phi(1-\phi(t))} = \frac{\phi(t)}{\phi(t)+\phi(1-\phi(t))}.
\end{aligned}$$

To determine $\phi(t)$ note that the chance of surviving to time t and remaining relaxed is $e^{-(\lambda+\alpha\phi)t}$. The chance of surviving to time t and being desperate is more complicated. For this to occur one must become desperate at some time $\tau \leq t$, and not sell, which occurs with probability:

$$\int_0^t e^{-\alpha\phi\tau}e^{-\alpha(t-\tau)}\lambda e^{-\lambda\tau}d\tau = \frac{\lambda e^{-(\lambda+\alpha\phi)t}\left(1-e^{-(\alpha(1-\phi)-\lambda)t}\right)}{\alpha(1-\phi)-\lambda}$$

Then, $\phi(t)$ equals the chance of surviving to time t and being desperate, divided by the chance of surviving to time t, which yields:

$$\phi(t) = \frac{1}{1+\dfrac{e^{\alpha t}(\lambda-\alpha(1-\phi))}{\lambda\left(e^{(\lambda+\alpha\phi)t}-e^{\alpha t}\right)}}$$

So, altogether we have:

$$\begin{aligned}
f(p(R,D)|t) &= (1-\phi)Pr(s=D|t) &&= \tfrac{(1-\phi)\phi(t)}{\phi(t)+\phi(1-\phi(t))} \\
f(p(D,D)|t) &= \phi\Pr(s=D|t) &&= \tfrac{\phi\phi(t)}{\phi(t)+\phi(1-\phi(t))} \\
f(p(D,R)|t) &= 1-Pr(s=D|t) &&= 1-\tfrac{\phi(t)}{\phi(t)+\phi(1-\phi(t))}.
\end{aligned}$$

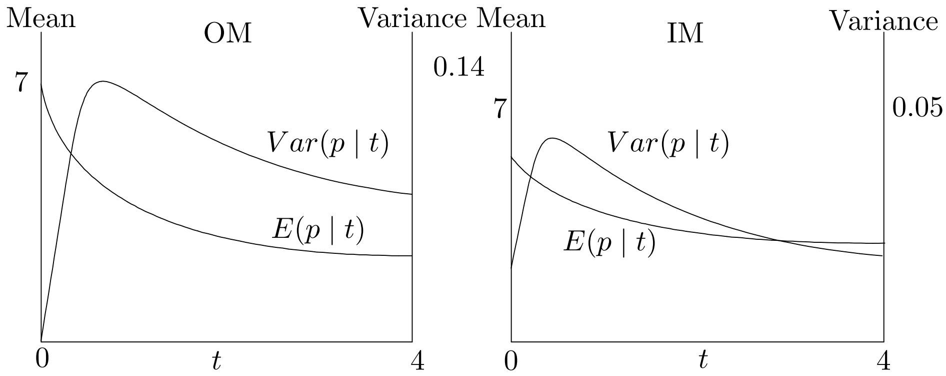

Figure 5: Mean and variance of the sales price conditional on time to sale for the case: $(\alpha, \lambda, r) = (2, 1, 0.1)$ and x at the threshold between the IM and OM cases (approximately 13.65). The scale for each case differs, in both cases the mean is measured along the left hand axis, while the variance is measured along the right hand axis.

Finally, we use these formulae to determine conditional means and variances. The exact expressions are complicated, and so we restrict their display to the Appendix. We summarize their characteristics in the following proposition. See Figure 5 for a parametric example.

Proposition 3 *In the IM and OM cases, the expected price falls with time to sale. In both cases the variance of price initially increases and then later falls with time to sale, specifically:*

IM *For all* $t < \dfrac{\log 2}{\lambda}$ *the variance is increasing in* t*, and decreasing thereafter.*

OM *The variance increases near* $t = 0$*, and there is a cutoff* τ *such that the variance decreases for all* $t > \tau$*.*

The result for expected price is clearly intuitive. The longer is the time to sale, the less likely the seller is to be relaxed. The reason that $Var(p)$ first increases and then decreases is that for low values of t, almost all sellers are relaxed, for high values of t, almost all sellers are desperate, and for intermediate values of t, seller types are mixed. Given that the distribution of buyer types is stationary, this mixing is what leads to the greater variance in sales price.

5 Remarks and Conclusions

In this paper, we present a simple matching model of the housing market. We assume agents are heterogeneous with respect to the flow values they retain if they remain unmatched but homogeneous in their match values in the sense that the value of a consummated transaction does not depend on the agents' types. We also assume that buyers and seller experience flow value transitions. This captures the idea that while buyers and sellers are relaxed upon entering the housing market, they eventually become desperate ("motivated") if they do not match in the meantime.

Our model has a number of implications. First, in contrast to the outcomes that one often sees in models with match value heterogeneity, strict positive assortative and negative assortative matching cannot occur in steady-state equilibrium. In the symmetric case, there are three matching patterns that can occur: (i) indiscriminate matching (IM) – all potential sales are consummated; (ii) opportunistic matching (OM) – desperate agents will buy from or sell to anyone, but relaxed agents wait to find desperate partners; and (iii) desperate matching (DM) – the only transactions that occur are those between desperate buyers and sellers. Fixing the other parameters of the model, we show that IM necessarily obtains if x, the value of a house, is sufficiently high. For lower values of x, OM is the unique equilibrium outcome; for still lower values, either OM or DM can obtain; for the lowest values of x, DM is the unique equilibrium outcome. We also completely characterize the equilibrium correspondence for the general (asymmetric) case.

We explore the implications of our model in the symmetric case in more detail. Specifically, we derive the marginal distribution of price, the marginal distribution of time to sale, and the conditional distribution of price given time to sale for the three equilibrium types (IM, OM, and DM). We analyze how expected price and price dispersion, both unconditionally and conditional on time to sale, vary with model parameters, both within and across equilibrium types. Similarly, we show how expected time to sale depends on model parameters.

Our objective in this paper has been to construct a simple foundation that can serve as a basis for more elaborate models of the housing market. As such, we have omitted many realistic details. In many cases, we believe these can be added relatively easily. One potential elaboration is to assume that x is an idiosyncratic draw from some distribution F as in Williams (1995) and Arnold (1999). The assumption of a match-specific draw might be a force in favor of NAM. A relaxed buyer can afford to wait for a particularly favorable draw from F On the other hand, a relaxed seller can afford to wait for a buyer who really loves his house. The assumption of a match-specific draw puts relaxed sellers in a better bargaining position, which in turn increases the incentive for a relaxed seller to wait for a desperate, and hence exploitable, partner. The other virtue of this assumption about x is its empirical potential. We do not observe one- or three-point price distributions, even for virtually identical houses. Adding a match-specific component would smooth the price distribution.

A second potential elaboration is to allow more heterogeneity across agents. For example, one might assume that there is a distribution of λ across new entrants to the market so that some agents expect to become desperate more quickly than others do. This opens up the possibility of a positive correlation between price and time to sale. The reason is that sellers with low values of λ can afford to wait for prospective buyers who either are already desperate or who are at high risk of becoming so. This is a force that biases the composition of prospective sellers whose houses have been on the market a long time towards those who have good bargaining positions. Of course, at the same time, heterogeneity in λ among buyers biases the composition of the pool on that side of the market towards those with low values of λ since those are the buyers who can afford to hold out for a good deal.

There are myriad other possible elaborations. Most of the ones that we think have substantial empirical potential can be incorporated into our framework without great analytical difficulty. This is the sense in which we argue that we have constructed a useful foundation for search-matching models of the housing market.

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Appendix

A Asymmetric Case Details

In this section we present the details behind Proposition 2 and the corresponding Figure 3. We do not explicitly solve for equilibrium prices (not required for the proposition), but could certainly do so using the value functions we derive. Recall that Figure 3 is drawn for the case of $\beta + \sigma + rx = 1$.

Indiscriminate Matching. The value functions in this case are

$$rB^R = \beta + \alpha\phi_s[x - p(R, D) - B^R] + \alpha(1 - \phi_s)[x - p(R, R) - B^R] + \lambda(B^D - B^R).$$

$$rB^D = \alpha\phi_s[x - p(D, D) - B^D] + \alpha(1 - \phi_s)[x - p(D, R) - B^D]$$

$$rS^R = \sigma + \alpha\phi_b[p(D, R) - S^R] + \alpha(1 - \phi_b)[p(R, R) - S^R] + \lambda(S^D - S^R)$$

$$rS^D = \alpha\phi_b[p(D, D) - S^D] + \alpha(1 - \phi_b)[p(R, D) - S^D]$$

When there is indiscriminate matching on both sides of the market, $\phi_s = \phi_b = \phi$. Taking this into account, substituting the Nash prices into the value functions, and solving simultaneously yields expressions for B^R, S^R, B^D, and S^D.

For this equilibrium to make sense, we need $x - B^b - S^s \geq 0$ for all b, s. Since relaxed types enjoy a higher payoff in equilibrium than desperate types, we simply need: $x - B^R - S^R \geq 0$. Substituting the expressions for B^R and S^R and simplifying, we find:

$$rx \geq \frac{(\beta + \sigma)(2r + \alpha(1 + \phi))}{(2r + 2\lambda + \alpha)}$$

Thus, in our diagram (in which we have fixed α, λ, and r) the IM region is defined by a line parallel to the $\beta\sigma$ side of the triangle. Solving for the $\sigma = 0$ intercept of this line (setting $\beta = 1 - rx$) yields:

$$rx = \frac{2r + \alpha(1 + \phi)}{4r + 2\lambda + \alpha(2 + \phi)}$$

Opportunistic Matching. The value functions in this case are

$$rB^R = \beta + \alpha\phi_s[x - p(R, D) - B^R] + \lambda(B^D - B^R).$$

$$rB^D = \alpha\phi_s[x - p(D, D) - B^D] + \alpha(1 - \phi_s)[x - p(D, R) - B^D]$$

$rS^R = \sigma + \alpha\phi_b[p(D,R) - S^R] + \lambda(S^D - S^R)$

$rS^D = \alpha\phi_b[p(D,D) - S^D] + \alpha(1-\phi_b)[p(R,D) - S^D]$

When there is opportunistic matching, $\phi_s = \phi_b = \phi$. Using this, substituting for prices, and solving simultaneously yields for B^R, S^R, B^D, and S^D.

For this equilibrium to make sense, we need $x - B^R - S^R \leq 0$ and $x - B^b - S^s \geq 0$ for all other b, s. Again relaxed agents have a higher value than their desperate counterparts, so we need not be concerned with the condition $x - B^D - S^D \geq 0$. We now substitute the value functions into each of these expressions in turn and simplify to determine the allowable ranges for x to support OM.

i) $x - B^R - S^R \leq 0$.

$$rx \leq \frac{(\beta+\sigma)(2r+\alpha+\alpha\phi)}{(2r+\alpha+2\lambda)}.$$

Note that this is precisely the opposite of the condition required for IM to be an equilibrium.

ii) $x - B^R - S^D \geq 0$.

$$rx \geq \frac{(2r+\alpha+\alpha\phi)\beta}{(2r+\alpha+2\lambda)} - \frac{\alpha^2\phi(\alpha+2\lambda)\sigma}{(2r+\alpha+2\lambda)\left[2r(\alpha+\lambda)+\lambda(3\alpha+2\lambda)\right]}. \tag{A1}$$

This defines a line running from the interior of the $rx\beta$ segment to the interior of the $\sigma\beta$ segment in Figure 3. Note that the $\sigma = 0$ intercept of this line is the same as the line defining the IM region, as drawn in Figure 3.

iii) $x - B^D - S^R \geq 0$.

This condition is the same as that immediately preceding with the roles of β and σ reversed. Thus, the $\beta = 0$ intercept is the same as that for the line defining the IM region.

Desperate Matching. The value functions in this case are easily determined to be

$$\begin{aligned} S^D = B^D &= \frac{\alpha\phi^D x}{2\left(r+\alpha\phi^D\right)} \\ B^R &= \frac{2\left(r+\alpha\phi^D\right)\beta + \alpha\phi^D x\lambda}{2\left(r+\alpha\phi^D\right)(r+\lambda)} \\ S^R &= \frac{2\left(r+\alpha\phi^D\right)\sigma + \alpha\phi^D x\lambda}{2\left(r+\alpha\phi^D\right)(r+\lambda)}. \end{aligned}$$

For desperate matching to be an equilibrium we need $x - B^D - S^D \geq 0$, while the inequality must be flipped for all other possible matched pairs. For now assume:

$$rx \leq \frac{2\beta(r+\alpha\phi^D)}{\alpha\phi^D} \quad \text{and} \quad rx \leq \frac{2\sigma(r+\alpha\phi^D)}{\alpha\phi^D}. \tag{A2}$$

so that, $B^R \geq B^D$ and $S^R \geq B^D$ (we shall verify that these conditions are not binding). Then we need only check that $B^R + S^D \geq x$ and $B^D + S^R \geq x$.
i) $B^R + S^D \geq x$**.** Substituting the value functions into the condition gives:

$$rx \leq \frac{2\beta(r+\alpha\phi^D)}{(2r+2\lambda+\alpha\phi^D)} \tag{A3}$$

Note that this is more restrictive than the first inequality in (A2), since the denominator is strictly larger. When it holds with equality (A3) defines a line which starts in the $\sigma = 1$ corner and intersects the $rx\beta$ line segment. We know from the $\sigma = \beta$ case that this line intersects the $\sigma = \beta$ line at a higher value of rx than the lines defining the upper boundary (RHS boundary) of the OM region. It turns out the $\sigma = 0$ intercept is at a lower rx value. To see this, one can easily solve for the intercept:

$$rx = \frac{2(r+\alpha\phi^D)}{4r+2\lambda+3\alpha\phi^D}$$

which is less than the $\sigma = 0$ intercept of the line defining the IM region, since

$$\begin{aligned}
\frac{2(r+\alpha\phi^D)}{4r+2\lambda+3\alpha\phi^D} &< \frac{2r+\alpha(1+\phi)}{4r+2\lambda+\alpha(2+\phi)} \\
\frac{2(r+\alpha\phi^D)}{2r+2\lambda+\alpha\phi^D+2r+2\alpha\phi^D} &< \frac{2r+\alpha(1+\phi)}{2r+2\lambda+\alpha+2r+\alpha(2+\phi)} \\
\frac{2r+2\lambda+\alpha\phi^D+2r+2\alpha\phi^D}{2(r+\alpha\phi^D)} &> \frac{2r+2\lambda+\alpha+2r+\alpha(2+\phi)}{2r+\alpha(1+\phi)} \\
\frac{2r+2\lambda+\alpha\phi^D}{2(r+\alpha\phi^D)}+1 &> \frac{2r+2\lambda+\alpha}{2r+\alpha(1+\phi)}+1 \\
\frac{2(r+\alpha\phi^D)}{2r+2\lambda+\alpha\phi^D} &< \frac{2r+\alpha(1+\phi)}{2r+2\lambda+\alpha}
\end{aligned}$$

which we established in the symmetric case (see Fig. 2).

ii) $B^D + S^R \geq x$. Substituting in value functions we find:

$$rx \leq \frac{2\sigma(r+\alpha\phi^D)}{(2r+2\lambda+\alpha\phi^D)}$$

Note that this is more restrictive then the second inequality in (A2), since the denominator is strictly larger. When this condition holds with equality it defines the lower of the two lines starting in the $\beta = 1$ corner, as shown in Figure 3.

Buyer Opportunistic Matching. When BOM obtains, relaxed sellers do not match, while desperate sellers match with anyone. Thus, relaxed buyers 'opportunistically' match with desperate sellers (see Figure 1 for a reminder). The seller opportunistic matching case is the same with the roles of buyers and sellers reversed, so we need only solve the BOM case here.

We did not solve for the steady state conditions in the text for this case, so we do so now. For sellers, the flow into the desperate state is $\lambda(1-\phi_s)$, while the flow out of the desperate state is $\alpha\phi_s$. For buyers, the flow into the desperate state is $\lambda(1-\phi_b)$, while the flow out is $\alpha\phi_b\phi_s$ (since desperate buyers will only match with desperate sellers). Thus, the steady state satisfies

$$\begin{aligned}\lambda(1-\phi_s) &= \alpha\phi_s \\ \lambda(1-\phi_b) &= \alpha\phi_b\phi_s\end{aligned}$$

which yields

$$\phi_b = \frac{\alpha+\lambda}{2\alpha+\lambda}, \qquad \phi_s = \frac{\lambda}{\alpha+\lambda}$$

For BOM to be an equilibrium we require

$$B^R + S^R \geq x;\ B^D + S^R \geq x;\ B^R + S^D \leq x;\ B^D + S^D \leq x.$$

Only the second and third constraint can be binding (again, relaxed agents have a higher value than desperate agents). In the interest of brevity, we suppress the specific formula for the value functions in this case, and instead go directly to the implied restrictions on x.

i) $B^D + S^R \geq x$. Substitution yields:

$$rx \leq \frac{(2r+\alpha+\alpha\phi)\sigma}{(2r+\alpha+2\lambda)} - \frac{\alpha^2\lambda(\alpha+2\lambda)\beta}{(\alpha+\lambda)(2r+\alpha+2\lambda)\left[2r(\alpha+\lambda)+\lambda(3\alpha+2\lambda)\right]}$$

Note that this is the opposite of inequality (A1), and thus the upper boundary for the BOM region is coincident with the boundary for the OM region.

ii) $B^R + S^D \leq x$.

$$rx \geq \frac{\beta(2r + \alpha(\phi_b + \phi_s))}{(2r + 2\lambda + \alpha\phi_s)}$$

When this condition holds with equality it defines the higher of the two lines that starts from the $\sigma = 1$ corner in Figure 3. Since BOM = OM when $\beta = \sigma$ this line must meet the peak of the OM region on the $\beta = \sigma$ line. Note that this means the BOM region must overlap the DM region.

Seller Opportunistic Matching. This case is identical to the Buyer Opportunistic Matching case, with the roles of β and σ reversed.

B Proof of Proposition 3

Using the formula displayed in the text, determining the following means and variances is a straightforward (if tedious) task:

$$\begin{aligned}
E_{IM}(p \mid t) &= \frac{x}{2} + \frac{(\alpha+\lambda)e^{-\lambda t} - \alpha}{(\alpha+\lambda)(2r+\alpha+2\lambda)} \\
Var_{IM}(p \mid t) &= \frac{\alpha\lambda + (\alpha+\lambda)^2(1-e^{-\lambda t})e^{-\lambda t}}{(\alpha+\lambda)^2(2r+\alpha+2\lambda)} \\
E_{OM}(p \mid t) &= \frac{x}{2} + \frac{h_1(\alpha,\lambda,r)\left[\alpha(\alpha+\lambda)e^{\frac{(\lambda^2+2\alpha\lambda)t}{\alpha+\lambda}} - \lambda(2\alpha+\lambda)e^{\alpha t}\right]}{h_2(\alpha,\lambda,r)\left[(\alpha+\lambda)^2 e^{\frac{(\lambda^2+2\alpha\lambda)t}{\alpha+\lambda}} - \alpha(2\alpha+\lambda)e^{\alpha t}\right]} \\
Var_{OM}(p \mid t) &= \frac{h_1(\alpha,\lambda,r)^2(\alpha+\lambda)(e^{\frac{(\lambda^2+2\alpha\lambda)t}{\alpha+\lambda}} - e^{\alpha t})(\alpha\lambda(\alpha+\lambda)e^{\frac{(\lambda^2+2\alpha\lambda)t}{\alpha+\lambda}} + h_3(\alpha,\lambda,r)e^{\alpha t})}{2h_2(\alpha,\lambda,r)^2\left[(\alpha+\lambda)^2 e^{\frac{(\lambda^2+2\alpha\lambda)t}{\alpha+\lambda}} - \alpha(2\alpha+\lambda)e^{\alpha t}\right]^2}
\end{aligned}$$

where:

$$\begin{aligned}
h_1(\alpha,\lambda,r) &= (\alpha+r)(\alpha^2 xr - 2\lambda r - 2\alpha(r+\alpha+\lambda) < 0 \\
h_2(\alpha,\lambda,r) &= 2r^2(\alpha+\lambda)^2 + \alpha\lambda(2\alpha^2 + 5\alpha\lambda + 2\lambda^2) + r(\alpha^3 + 6\alpha^2\lambda + 7\alpha\lambda^2 + 2\lambda^3) > 0 \\
h_3(\alpha,\lambda,r) &= \lambda^3 + 4\alpha\lambda^2 + 2\alpha^2\lambda - 4\alpha^3.
\end{aligned}$$

It is not immediately obvious why h_1 must be negative. To see that it must be, first note that $h_1 < 0$ *if and only if* $x > 2(\alpha+\lambda)(\alpha+r)/r\alpha^2$. If we can show that this threshold is greater then the upper bound of the OM range then we are done, for OM cannot obtain for such high x. Thus, if we need to show that:

$$\frac{(\alpha+\lambda)(\alpha+r)}{\alpha^2} > \frac{(2r+\alpha+\alpha\phi)}{(2r+2\lambda+\alpha)}.$$

If we cross multiply, then we have:

$$(\alpha+\lambda)(\alpha+r)(2r+2\lambda+\alpha) - (2r+\alpha+\alpha\frac{\lambda}{\alpha+\lambda})\alpha^2 > 0$$

which if expanded is clearly positive.

One can verify that $\partial E_{IM}(p \mid t)/\partial t < 0$ by inspection. The partial derivative of $Var_{IM}(p \mid t)$ is of the same sign as:

$$\frac{\partial(1-e^{-\lambda t})e^{-\lambda t}}{\partial t} = \lambda e^{-2\lambda t}(2-e^{\lambda t})$$

so that the threshold τ solves: $e^{\lambda\tau} = 2$, i.e. $\log 2/\lambda$.

For the OM case we calculate:

$$\frac{\partial E_{OM}(p \mid t)}{\partial t} = \frac{h_1(\alpha,\lambda,r)(2\alpha+\lambda+)(\lambda^2+\alpha\lambda-\alpha^2)^2 e^{\frac{((\alpha+\lambda)^2+\alpha\lambda)t}{\alpha+\lambda}}}{2h_2(\alpha,\lambda,r)\left[(\alpha+\lambda)^2 e^{\frac{(\lambda^2+2\alpha\lambda)t}{\alpha+\lambda}} - \alpha(2\alpha+\lambda)e^{\alpha t}\right]^2} < 0$$

In order to establish the result for the variance in the OM case, we calculate:

$$\frac{\partial Var_{OM}(p \mid t)}{\partial t} = -e^{(\alpha+\lambda+\frac{\alpha\lambda}{\alpha+\lambda})t}\left[h_1(\alpha,\lambda,r)^2(\alpha^2-\alpha\lambda-\lambda^2)^2\right]\Psi(t)$$

where

$$\Psi(t) = \frac{(\alpha+\lambda)(4\alpha^2+3\alpha\lambda+\lambda^2)e^{\frac{\lambda(2\alpha+\lambda)t}{\alpha+\lambda}} - \lambda(2\alpha+\lambda)(5\alpha+2\lambda)e^{\alpha t}}{\left[(\alpha+\lambda)^2 e^{\frac{\lambda(2\alpha+\lambda)t}{\alpha+\lambda}} - \alpha(2\alpha+\lambda)e^{\alpha t}\right]^3}$$

First note that this derivative is continuous, and evaluating this derivative at $t = 0$ yields:

$$\frac{\partial Var_{OM}(p \mid 0)}{\partial t} = \frac{h_1(\alpha,\lambda,r)^2(4\alpha+\lambda)}{2h_2(\alpha,\lambda,r)^2} > 0$$

so the derivative is positive near $t = 0$. Now note that as $t \to \infty$ the numerator and denominator of $\Psi(t)$ are dominated by the same term (either $e^{(\lambda+\frac{\alpha\lambda}{\alpha+\lambda})t}$ or $e^{\alpha t}$), thus for t high enough the numerator and denominator are of the same sign, so the whole expression is negative (actually, it tends to 0, but from below). $\square$