Abstract

We present a simple and tractable trade model of heterogeneous firms, endogenous quality choice, and endogenous markups. A key feature of the model is that competition not only lowers the cost threshold between the firms that produce and those that exit, but it also raises the scope for quality differentiation. With both these channels present, the most productive firms respond to competition by raising quality, prices, and markups, while the least productive either exit or respond in the exact opposite manner.

The model generates a unified theory and a supply-side explanation for an extremely rich set of stylized facts relating to (i) firm heterogeneity, (ii) product quality heterogeneity, (iii) markups heterogeneity, (iv) heterogeneity in the response of firms to competition, and (v) heterogeneity in the sign and magnitude of the correlations between output prices, firm productivity, size, and product quality. In addition, the model predicts that average price and markups exhibit a U-shape response to competition, that imports from developed countries are of higher quality and cost more than imports from developing countries, and that welfare gains are significantly understated when trade-induced competition is accounted for, but trade-induced innovation is not.

**Keywords:** Intra-industry trade, firm heterogeneity, quality choice, non-constant markups, quality ladders

**JEL classification:** F12, O14, O47
1 Introduction

The availability of firm- and plant-level data spurred the development of a new trade literature that put firm heterogeneity at its core. The seminal work of Bernard et. al. (2003) and Melitz (2003) provided economists with new intuition as to why more productive firms have higher market shares, are more likely to export, serve more export destinations, and account for most of a nation’s exports.\(^1\)

Subsequent empirical work emphasized that in addition to firm heterogeneity, quality choice by firms is critical in explaining patterns of trade.\(^2\) Most importantly, a set of studies argued that observed positive correlations between export prices and productivity (Verhoogen 2008, Baldwin and Harrigan 2011), between output prices and market shares (Verhoogen 2008, Kugler and Verhoogen 2011, Manova and Zhang 2012), and observed patterns between export prices and destination market characteristics (Bastos and Silva 2011, Manova and Zhang 2012, Kneller and Yu 2008) could not be accounted for without considering firms’ quality choices.

Among the first economists to introduce quality choice into heterogeneous trade models, Johnson (2012), Verhoogen (2008), Baldwin and Harrigan (2011), and Kugler and Verhoogen (2011) expanded the Melitz (2003) model of heterogeneous firms and constant markups by considering how firms optimally choose quality.\(^3\) These models produced quality sorting across firms, where firms endowed with low marginal costs (higher productivity) produce high quality goods, while firms endowed with high costs (low productivity) produce low quality goods.

Although augmenting heterogeneous firms models with endogenous quality choice constitutes a major

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\(^1\) For evidence see Eaton et. al. (2004, 2008,) and Bernard et. al. (2009).

\(^2\) The literature on the importance of quality goes as far back as Linder (1961) who observed that rich countries produce and consume a higher share of quality goods. Schott (2004), Hummels and Klenow (2005) and Fontagne, Gaulier and Zignano (2008) documented that in developed countries, large portions of exports increases occur through quality upgrades. Bernard et al (2006) and Hallak (2006) showed that capital- and skill-abundant countries use their endowment advantages to produce vertically superior varieties that have higher prices and higher quality. In addition, Verhoogen (2008) attributed the rising wage inequality gap in Mexican manufacturing firms in the 90s to the demand for high quality products by rich countries, which raised the relative demand for skilled labor.

step forward, the assumption that markups are constant across firms limits the models’ empirical relevance. Empirical evidence show that markups are heterogeneous across firms, increase in firm productivity and size, and are higher for exporters, even when controlling for productivity and market size (Roberts and Supina 2000, De Loecker and Warzynski 2012). In addition, there is evidence that high-quality goods command higher markups and that firms adjust markups in response to changes in demand conditions, which can occur over business cycles, when competition toughens, or when exchange rates move.4

We address this shortcoming, and thereby provide a unified, supply-side explanation of the three sets of stylized facts discussed above - relating to firm heterogeneity, quality heterogeneity, and markup heterogeneity – by introducing endogenous quality in the Melitz-Ottaviano (MO, 2008) model of linear demand systems and endogenous markups.5

The model produces a quality ladder where firms are placed according to their marginal cost. The lower the marginal cost is for a firm, the higher the optimal quality it chooses for its product, the lower its elasticity of substitution, and the higher the markup. It is helpful to visualize the quality ladder as a ladder placed against a wall, with the wall being the quality axis.

The slope of the ladder also affects the firm’s quality choice. While the marginal cost pins down the firm’s position on the ladder, the scope for quality differentiation in a given market pins down the slope. When the scope for quality differentiation changes, the slope of the ladder changes as well.

In addition to incorporating firm, quality, and markup heterogeneity in a simple and tractable model, another major contribution of this work is the rich set of predictions it generates on issues that extend beyond the areas discussed above. It allows firms to respond heterogeneously to competition and it resolves a set of inconsistencies in the literature.

Economists have long been interested in the relation between competition and innovation. In a series of influential papers, Aghion et al. (1997, 2001, 2005) argued that the relation between the two is not linear:


5 Furthermore, as we show in Section 2, the theory offers a supply-side explanation for the Linder hypothesis.
firms at the technological frontier respond to competition by raising quality-improving innovation, whereas firms below the technological frontier innovate less than they did before.

Recent empirical studies provide ample support for their view. These studies show that more productive firms respond to competition by investing more in innovation, by investing in better technologies, and by upgrading product quality, while the least-productive firms respond in just the opposite ways. Even though the non-linear response of firms to competition is now well documented, existing trade models of heterogeneous firms assume a linear response and thus fail to produce results consistent with the aforementioned observations.

Our model addresses firms’ heterogeneous response to competition by linking the scope for quality differentiation to market toughness. An increase in market toughness raises the scope for quality differentiation, and causes the ladder to pivot around some point on the ladder. Firms above that point, the more productive ones, escape competition by raising quality, markups and prices, while firms below the pivot point either elect to lower quality, markups, and prices, or they exit. The model is thus consistent with the fourth set of stylized facts presented above on firms’ heterogeneous response to competition. It is also consistent with Sutton’s (1989, 1991) observation that market competition does not always result in market fragmentation. Indeed, the market shares of firms that escape competition by innovating rise.

6 Amiti and Khandelwal (2009) document that in the US, trade liberalization resulted in quality upgrading for products close to the word quality frontier, and discouraged quality upgrading for products far from the frontier. Bloom et al. (2011) found that high total-factor-productivity (TFP) firms in the EU were more likely to respond to the increased import competition brought by China’s entry to the WTO by innovating, than low TFP firms were. Similarly, using Mexican data before and after NAFTA, Iacovone (2012) found that liberalization boosted innovation efforts by more productive firms while it weakened the incentive to innovate for less productive firms (Iacovone 2009). Similarly, Lileeva and Treffer (2010) and Shor (2004) showed that the impact of trade liberalization is heterogeneous across firms. Using a panel of British manufacturing firms, Blundell and Griffith (1999) documented that increased competition tends to simulate innovation by the dominant firms. Finally, Bustos (2011) and Teshima (2010) use very detailed plant-level data from Argentina and Mexico to document firms’ investments in technology and innovation in response to trade liberalization.

7 An exception is Bustos (2011). The author provides a model where more productive firms respond to trade liberalization by upgrading to better technology, while all other firms continue to use the old, and more costly, technology. Her work is a good example of a non-linear, but binary, response of firms to competition.

8 For a graphical representation see Figure 2. Quality ladders (and the appropriate figures) are introduced in detail in the next section.
To understand why changes in competition affect the scope for quality differentiation, consider the effects of an increase in market size on the economy. Consistently with a large set of models, an increase in market size increases competition as it encourages more firms to enter the market, and as a consequence, it lowers the cost threshold that distinguishes firms that continue to operate in the market from those that exit.

But there is now a second channel through which competition affects incumbent firms and potential entrants. A larger market raises the scope for quality differentiation because it makes it easier for firms to recover the fixed cost of innovation. Since the slope of the quality ladder in the model is equal to the scope for quality differentiation, the slope of the ladder increases. These two effects, namely the decrease in the cost threshold and the increase in the scope for quality differentiation cause the ladder to pivot around a point on the ladder, which then generates a heterogeneous response of firms to competition. The (endogenous) relation between market toughness and the scope for quality differentiation is a key element of the model and constitutes an important departure from past work.

The model also provides clarity on the relation between prices, productivity, market shares, and quality. In heterogeneous firms trade models, firm size is positively related to firm productivity. Models that do not incorporate quality predict a negative correlation between prices and productivity, and therefore between prices and firm size. However, if quality is incorporated in these models, and if its production raises marginal costs substantially, then the correlation between prices and productivity, and between prices and firm size becomes positive. Since these models produce quality sorting along the productivity axis, then the correlation between prices and quality is also positive. Put differently, prices are good proxies for quality.

Yet empirical evidence suggests that the correlations between prices and productivity, and between firm size and productivity are positive in some industries but negative in others, and that prices are not always a good proxy for quality. Roberts and Supina (1996, 2000), Syverson (2007), and Foster, Haltiwanger, and Syverson (2008) report a negative correlation between firm size and output price, while Verhoogen (2008), Kugler and Verhoogen (2011), Hallak and Sivadasan (2011), Manova and Zhang (2012), and Iacovone and Javorcik (2012) report positive correlations.

\[\text{For a step-by-step exposition, see Baldwin and Harrigan (2011).}\]
\[\text{Even the evidence in favor of a positive correlation between prices and productivity reported in Baldwin and Harrigan (2011) is not conclusive. The authors regress US export unit prices on a series of gravity variables, including distance, and}\]

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price is a good proxy for quality but not in others.

Variation in the scope for quality differentiation across industries helps explain these inconsistencies. When the scope for quality differentiation is high, the quality ladder is steep (long ladder). High productivity firms choose high quality, which raises markups substantially. Although marginal costs fall in this model as firm productivity increases, the increase in markups offsets the falling costs, and prices increase as firm productivity, size, and quality increase.

However, when there is little scope for quality differentiation, although high productivity still translates to higher product quality, the ladder is relatively flat (short ladder) and the increase in markups is thus not sufficient to offset falling costs. Therefore, prices decline, albeit not as fast as in a model without quality choice, as firm productivity, size, and product quality increase.

Indeed, Khandelwal (2010) finds that prices are a good proxy for quality for long quality ladders, but not for short. Moreover, Kugler and Verhoogen (2011) argue that the positive correlations they observe in their data is because they consider heterogeneous sectors with higher scope for quality differentiation, whereas the studies that find negative correlations consider homogeneous sectors. In further support to the intuition presented in this model, Kugler and Verhoogen (2011) show that the correlation between prices and firm size increases with the scope for quality differentiation (proxied by R&D and advertising spending). And using Chinese data, Manova and Zhang (2012) also confirm that the correlation between export prices and firm revenues increases substantially when the scope for quality differentiation in a given industry is high.

To summarize, the model provides a supply-side explanation for an extremely rich set of facts related to (i) firm heterogeneity, (ii) product quality heterogeneity, (iii) markups heterogeneity, (iv) heterogeneity in the response of firms to competition, and (v) heterogeneity in the size and magnitude of the correlations between output prices, firm size, and product quality. This is achieved by endogenizing quality choice, by argue that the positive relation they observe between export prices and distance provides proof of a positive correlation between prices and productivity. However, a simple replication of their regression specification across industries reveals that in many industries - 34% and 31% of all the SITC5 and HS6 manufacturing industries, respectively – the distance coefficient comes up negative, which is evidence that in those industries prices and productivity are negatively correlated. For further evidence that the relation between export unit prices and distance can be either positive or negative across industries and across trading partners, see Kneller and Yu (2008) and Manova and Zhiang (2012), respectively.
endogenizing markups, and by letting market toughness affect the scope for quality differentiation. Despite
the richness of the theory, the model is simple and tractable.

In addition to matching these sets of stylized facts, the model makes some novel predictions about
aggregate economic measures. It predicts that average price and markups exhibit a U-shape response to
competition, that imports from developed countries are of higher quality and cost more than imports from
developing countries, and that welfare gains for developed countries are significantly understated when trade-
induced competition is accounted for, but not trade-induced innovation.

The remainder of the paper is structured as follows. In Section 2 we introduce the closed economy
version of the model. We describe the construction of the quality ladders and we analyze the impact of
market toughness on competition and innovation. We account for the various heterogeneities and we then
consider the aggregate implications of the model. In section 3 we present the open economy version and
highlight some of the key implications that differ from past work. Section 4 concludes.

2 Closed Economy Model

We formalize our theory by introducing endogenous quality choice in the Melitz-Ottaviano (MO, 2008)
model of linear demand preferences and endogenous markups. On the demand side, we augment the demand
function to include quality.\textsuperscript{11} On the supply-side, we follow Shaked and Sutton (1983, 1987, 1990) and model
quality choice as an endogenous sunk cost that firms have to pay.\textsuperscript{12}

\textsuperscript{11}Foster, Haltiwanger, and Syverson (2008) consider an almost identical demand system. The emphasis of their work is
to document differences between physical and revenue productivity, and they treat quality (which they call taste shifter) as
exogenous.

\textsuperscript{12}Kugler and Verhoogen (2011) also discuss a production specification where firms have to pay a sunk cost in order to produce
quality, but assume markups are constant. Their objective is to investigate how marginal costs change with quality upgrades
as the cost of inputs rises.
2.1 Consumers

The preferences of a typical consumer are given by

\[ U = q_c^0 + \alpha \int_{i \in \Omega} q_i^c \, di + \alpha \int_{i \in \Omega} z_i \, di - \frac{1}{2} \gamma \int_{i \in \Omega} (q_i^c)^2 \, di - \frac{1}{2} \gamma \int_{i \in \Omega} (z_i)^2 \, di + \gamma \int_{i \in \Omega} (q_i^c z_i) \, di - \frac{1}{2} \eta \left\{ \int_{i \in \Omega} \left( q_i^c - \frac{1}{2} z_i \right) \, di \right\}^2 \]  

(1)

where \( q_c^0 \) and \( q_i^c \) represent the individual’s consumption of the numeraire good and each variety \( i \), respectively. The quality of each variety is given by \( z_i \). If all firms choose no quality (\( z_i \equiv 0 \)), the preference relation is identical to that in MO. The parameters \( \alpha \) and \( \eta \) capture the degree of substitution between each variety and the numeraire, and the parameter \( \gamma \) captures the degree of differentiation among the varieties. All parameters are assumed to be positive. The inverse demand for each variety is given by

\[ p_i = \alpha - \gamma q_i^c + \gamma z_i - \eta Q^c \]  

(2)

where \( Q^c = \int_{i \in \Omega} (q_i^c - \frac{1}{2} z_i) \, di \). By inverting (2) we obtain the demand for each variety consumed

\[ q_i = \frac{\alpha L}{\eta N + \gamma} - \frac{L}{\gamma} p_i + L z_i + \frac{\eta N L}{\gamma (\eta N + \gamma)} \bar{p} - \frac{1}{2} \frac{\eta N L}{\eta N + \gamma} \bar{z} \]  

(3)

where \( N \) is the number of varieties consumed, \( L \) is the size of the country, \( \bar{p} = (1/N) \int_{i \in \Omega^*} p_i \, di \), \( \bar{z} = (1/N) \int_{i \in \Omega^*} z_i \, di \), and \( \Omega^* \subset \Omega \) is the subset of varieties consumed. This specified preference specification ensures that the demand for good \( i \) is linear in price and quality.

2.2 Firms

As in Melitz (2003) and MO (2008), labor is the only factor of production. Firms pay a fixed market entry fee, \( f_E \), and then they draw a productivity parameter that determines their marginal cost, \( c \). The distribution of \( c \) is \( G(c) \) with support on \([0, c_m]\). Firms with high marginal cost (low productivity) exit the market. The remaining firms maximize profits by taking the number of firms \( N \), the average price \( \bar{p} \), and the average level of quality upgrade \( \bar{z} \) as given. Firms also choose the optimal level of quality upgrade. The
The cost function of a surviving firm \( i \) is given by

\[
TC_i = c_i q_i + \theta (z_i)^2
\]

(4)

The first term comes straight from MO (2008) and captures the variable cost of production. The second term captures the cost of quality upgrading, which is invariant to output. We assume that the cost of quality upgrading is convex. The parameter \( \theta \) is a key parameter; it captures country- or industry-specific differences in the ability to innovate or in the technology of innovation. In markets where the ability is high, the cost of innovation \( \theta \) is low. We can use variation in market size \( L \), to classify countries as small and large, and use variation in the cost of innovation \( \theta \), to classify countries as developed and developing, by considering how the ability to innovate differs across these types of countries.\(^{13}\)

The firms first set the output price as a markup over the marginal cost for a given level of quality upgrade.\(^{14}\) Let \( c_D \) be the marginal cost threshold between the firms that produce and the firms that exit. The firm with marginal cost \( c_D \) earns zero profits and its demand \( q(c_D) \) is driven to 0. Following MO, we can express all performance measures as functions of \( c, c_D, \) and \( z \).

\[
q(c, z) = \frac{L}{2\gamma} [c_D - c] + \frac{L}{2} z
\]

\[
p(c, z) = \frac{1}{2} [c_D + c] + \frac{\gamma}{2} z
\]

\[
r(c, z) = \frac{L}{4\gamma} [(c_D)^2 - c^2] + \frac{L}{2} z c_D + \frac{L\gamma}{4} z^2
\]

\[
\pi(c, z) = \frac{L}{4\gamma} [c_D - c]^2 + \frac{L}{2} z [c_D - c] + \frac{L\gamma}{4} z^2 - \theta z^2
\]

(5a, 5b, 5c, 5d)

Next, the firm chooses the optimal quality upgrade by maximizing the profit function (5d) above. The optimal quality \( z^* \) is

\[
z^* = \lambda (c_D - c)
\]

where \( \lambda = L/(4\theta - L\gamma) \). To ensure that \( z \) is positive, we assume that \( \lambda > 0 \) or that \( 4\theta > L\gamma \).

The relation above is extremely important. Firstly, it states that the optimal product quality is a function of two components: the scope for quality differentiation \( \lambda \), and the productivity of the firm \( c \), relative to the

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\(^{13}\)For evidence on the technological gap between developed and developing countries, see Treffer (1993), Hall and Jones (1999), Harrigan (1999) and Acemoglu and Zilibotti (2001).

\(^{14}\)For a given level of output, the firm chooses price and quality simultaneously in order to minimize the cost function. Given the linearity and separability of the model, we can first solve for the optimal price and then for the optimal level of quality.
cost threshold $c_D$. Secondly, it defines the scope for quality differentiation, which is the slope of the quality ladder. The scope depends on the market size $L$, the cost of innovation $\theta$, and the degree of differentiation among varieties $\gamma$. In larger, more differentiated markets, with lower cost of innovation (higher ability to innovate), the scope for quality differentiation is higher because it is easier for firms to recover the fixed cost of innovation.

The versatility of the model comes from the fact that we can express all economic measures as functions of the cost threshold $c_D$ and the scope for quality differentiation $\lambda$. While past work only considers how changes in the cost threshold affect the economy, we are able to match a richer set of stylized facts by considering how both $c_D$ and $\lambda$ respond to economic conditions. It is precisely this property of the model that generates the various heterogeneities and leads to novel theoretical predictions on economic aggregates that we discuss next.

Before we illustrate the quality ladders, we can re-write the performance measures as functions of $c$ and $c_D$.

\begin{align*}
    p(c) &= \frac{1}{2}(c_D + c) + \frac{\gamma\lambda}{2}(c_D - c) \\
    q(c) &= \frac{L}{2\gamma}(1 + \gamma\lambda)(c_D - c) \\
    r(c) &= \frac{L}{4\gamma}[(c_D)^2 - c^2] + \frac{L\lambda}{2} [c_D - c] c_D + \frac{L^2\gamma^2}{4} [c_D - c]^2 \\
    \pi(c) &= \frac{L}{4\gamma}(1 + \gamma\lambda)[c_D - c]^2 \\
    \mu(c) &= \frac{1}{2}(1 + \gamma\lambda)(c_D - c)
\end{align*}

In a setting with endogenous quality choice the scope for quality differentiation scales up prices, quantities, markups, and profits. Since in such a setting competition affects both $c_D$ and $\lambda$, its impact on the variables above will vary across firms.

### 2.3 Quality Ladders

The essence of the model is in the construction of the quality ladders and in the dynamics that characterize them. To fix ideas consider two sectors with different scopes for quality differentiation $\lambda$: sector 1 with low scope and sector 2 with high. Call sector 1 *homogeneous* and sector 2 *heterogeneous*. Furthermore, consider
two firms with identical costs $c_1$ and $c_2$, respectively, operating in each sector.\textsuperscript{15}

The solid line in Figure 1 depicts the quality ladder in the homogeneous (left diagram) and the heterogeneous (right diagram) sector. A ladder represents the optimal quality choice by all operating firms. We highlight three important observations. First, the quality ladder in the heterogeneous sector is steeper, since the scope for quality differentiation is higher. Second, the cost threshold in the heterogeneous sector is lower, which indicates that when the scope for quality differentiation is higher, competition is tougher. Third, a firm endowed with cost $c$ operating in the heterogeneous sector does not necessarily choose a higher level of quality when compared to an identical firm operating in the homogeneous sector. This can be seen from the graph by comparing the quality choice of firms endowed with cost $c_1$ and $c_2$ across the two sectors. We come back and explain this last observation shortly.

In both industries, more productive firms (move from right to left on the cost axis) face lower marginal costs, choose higher product quality, set higher markups, and are larger. But because markups and marginal costs move in opposite directions, higher productivity does not always translate to higher price. When the scope for quality differentiation is high (i.e. $\lambda > c_D/\gamma$), prices increase in productivity because markups are substantially large. However, when the scope is low, prices fall.

The relation between output prices and productivity across the two sectors is illustrated by the gray line in Figure 1. When the scope for quality differentiation is low (left diagram), prices are negatively correlated with firm productivity, negatively correlated with firm size, and negatively correlated with quality.\textsuperscript{16} When the scope is high (right diagram), prices are positively correlated with firm productivity, positively correlated with firm size, and positively correlated with quality. Hence, consistently with empirical evidence, the scope for quality differentiation across sectors determines both the sign and the magnitude of these correlations.

That is, in long quality ladders prices are a good proxy for quality but not in short ladders (Khandelwal 2010). Prices are negatively correlated with firm size in homogeneous sectors (Roberts and Supina 1996, \textsuperscript{15}While we discuss the intuition of the model in the context of sectors, the same intuition applies if you consider differences in the scope for quality differentiation across countries. Such differences can arise for example, if the market size or if the ability to innovate vary across countries. Size $L$, helps us differentiate between small versus large economies, and the cost of innovation $\theta$, between developing and developed. \textsuperscript{16}While we do not depict firm size on the same diagram, it is easy to see from equations 7c and 7d that firm size, either measured by revenue or by profit, increases in firm productivity.}
2000; Syverson 2007; and Foster, Haltiwanger, and Syverson 2008) but they are positively correlated in heterogeneous sectors (Verhoogen 2008; Kugler and Verhoogen 2011; Hallak and Sivadasan 2011; Manova and Zhang 2012; and Iacovone and Javorcik 2012). And the correlation between prices and firm size increases as the scope for quality differentiation rises (Kugler and Verhoogen 2011; Manova and Zhang 2012).

Next we show that market toughness affects firms in heterogeneous ways. This is illustrated in Figure 2. Market toughness, which is linked to an increase in market size $L$ and/or an increase in the ability of firms to innovate (decrease in $\theta$) operates through two channels: First, market toughness raises competition and lowers the cost threshold between the firms that operate in the economy and those that exit. Second, market toughness raises the scope for quality differentiation because it makes it easier for firms to recover the fixed cost of innovation associated with quality production. While the former channel is standard in the literature, the latter is a major departure and constitutes an innovation in the model.

In such a setting, as market toughens, the quality ladder pivots clockwise. The cost threshold falls, causing the least productive firms to exit (Area C). From the firms that remain in business, the least productive ones respond to competition by lowering quality, markups, and prices (Area B). In contrast, the more productive ones respond to competition by raising quality, markups, and prices (Area A). Market shares rise for the firms that respond to competition by raising quality-improving innovation, but fall for all others firms.

By endogenizing quality choice, markups, and the scope for quality differentiation, we are therefore able to come up with a unified theory and a supply-side explanation for a rich set of stylized facts related to 
(i) firm heterogeneity, (ii) product quality heterogeneity, (iii) markups heterogeneity, (iv) heterogeneity in the response of firms to competition, and (v) heterogeneity in the sign and magnitude of the correlations between prices, productivity, size, and quality.

2.4 Free-Entry Equilibrium

We close the model by considering the free-entry condition. In equilibrium, firms expect zero profits. Therefore

\[
f_E = \int_0^{c_d} \pi(c, z) dG(c) = \frac{L}{4\gamma} \int_0^{c_D} [c_D - c]^2 dG(c) + \frac{L}{4} \int_0^{c_s} [c_D - c]^2 dG(c) - \int_0^{c_s} f_z dG(c) \tag{7}
\]
The condition above determines the cutoff cost $c_D$. The number of surviving firms can be found from (2).

Set $q_i = 0$, then

$$c_D = \frac{1}{\eta N + \gamma} (a \gamma + \eta N \bar{p} - \frac{1}{2} \eta N \gamma \bar{z}) \quad (8)$$

It can be shown that

$$N = \frac{2 \gamma (a - c_D)}{\eta (c_D - \bar{c})} \quad (9)$$

The expression above is identical to the MO model without quality choice. However, in the endogenous quality version of the model the cost threshold is lower because competition is now tougher. And since the cost threshold is lower, the number of firms is higher.

### 2.5 Aggregate Predictions

Above, we have seen how firms optimize and how they respond to competition. Here we consider how these choices by the firms operating in a setting that is characterized by endogenous markups and endogenous quality affect aggregate variables in the economy. To do so, we parameterize the cost distribution.

Following Melitz (2003) and Melitz and Ottaviano (2008), suppose that the cost draws come from a Pareto distribution given by $G(c) = \left( \frac{c}{c_M} \right)^k, c \in [0, c_M]$. The cost threshold in the closed economy is

$$c_D = \left[ \frac{\gamma \phi}{(1 + \gamma \lambda)^2 L} \right]^{\frac{1}{k+2}} \quad (10)$$

where $\phi = 2 \epsilon_M^k (k + 1)(k + 2) f_E$. This cost threshold is a measure of competition within the economy. The lower $c_D$ is, the more competition exists. As the scope for quality differentiation rises, the cost threshold falls.

The average marginal cost in the economy is

$$\bar{c} = \frac{kc_D}{k + 1} \quad (11)$$

The higher the average marginal cost is, the lower the average productivity. The average quality in the
Given average productivity \( \bar{c} \) and the cost threshold \( c_D \), we can use (9) to solve for the number of firms.

\[
N = \frac{2(k+1)\gamma (\alpha - c_D)}{n} \tag{13}
\]

The average (unweighted) price in the economy is given by

\[
\bar{p} = \left( \frac{2k+1+\gamma \lambda}{2k+2} \right) c_D \tag{14}
\]

Average markups are

\[
\bar{\mu} = \frac{1}{2}(1+\gamma \lambda) \frac{1}{k+1} c_D \tag{15}
\]

Average profits are given by

\[
\bar{\pi} = f_E \left( \frac{c_M}{c_D} \right)^k \tag{16}
\]

Finally, welfare is

\[
U = 1 \frac{1}{2n}(\alpha - c_D) \left[ \alpha - \frac{k+1}{k+2} c_D \right] + \gamma \lambda (\alpha - c_D) \left[ \frac{12\alpha - 2(c_D - 3\alpha)k - (2 + 3\gamma \lambda)c_D}{2(k+2)n} \right]
\]

Aggregate measures of the economy are now functions of the cost threshold and the scope for quality differentiation. Competition affects the aggregate measures through two channels: it lowers the cost threshold and it raises the scope for quality differentiation. Because through the first channel competition suppresses quality, markups, and prices, while through the second it raises them, the total effect of competition on these measures will be ambiguous.

We illustrate that there is a U-shape response of prices, and markups to competition by considering how two economies identical in all aspects but in the cost of innovation \( \theta \), respond to an increase in market size. We use the increase in market size as proxy for competition. One economy, call it developing, has high cost of innovation, and the other, call it developed, has low cost. Figure 3 plots the cost threshold, the number of firms, the scope for quality differentiation, the average quality, price, and markups, and consumer welfare in each economy as market size increases. A solid black line represents the develop economy and a dash...
black line the developing. For comparison purposes, we also consider an economy where there is no scope for quality differentiation, such as the one modeled in MO. That economy is represented by a gray line.

As the market size increases, more firms enter the market and competition becomes tougher. Average productivity rises. In the developed economy, competition is much tougher, which results in a lower cost threshold, higher productivity (lower average cost), and a higher number of firms at each level of the competition.

As the level of competition increases, the scope for quality differentiation rises and so does average quality. In the developed economy the average quality of goods produced (and consumed) is higher than in the developing economy. This observation is known as the Linder hypothesis. While past work offers a demand-side explanation to the hypothesis based on the assumption that consumers in developed countries have higher preferences for quality goods (Verhoogen 2008; Linder 1961; Hallak 2008, Fajberbaum et. al. 2011), we show that supply-side effects based on differences in the scope for quality differentiation across countries offer an alternative explanation.

Next, we consider how average markups and average price respond to competition. Initially, both average markups and price fall. However, as competition increases further and product quality rises, average markups rise. Eventually, the increase in average markups pulls up average price. The U-shape respond of average markups and price to competition is a novel prediction of this model.\textsuperscript{17}

We conclude this section with a very important observation. Welfare increases substantially for the developed country as competition rises. This may come as a surprise. Welfare gains are typically associated with lower markups resulting from increased competition or trade. Yet, in the developed country average markups and price eventually rise, so intuitively we should expect welfare to fall. What happens here is that consumers like to pay low prices but they also like quality. And although prices and markups start to rise in the developed country after some level of competition, quality rises much faster, which ends up raising consumer welfare substantially. What the model reveals is that ignoring to account for the fact that consumers like quality understates welfare gains associated with competition and trade, and that this bias is

\textsuperscript{17}In the appendix we show that the variance of the firm performance measures is also a function of the cost threshold and the scope for quality differentiation. The response of price and markup dispersions to competition also exhibits a U-shape behavior.
larger for more developed countries.\textsuperscript{18} Put differently, developed countries that are characterized by a higher scope for quality differentiation stand to gain more from increases in market and competition - as happens in the case of trade, for example – whereas developing/poor countries gain little.

\section{Open Economy Model}

\subsection{Consumers}

We extend the closed economy model to a two-country setting. There is a home (H) and a foreign (F) country. Each country is endowed with $L^H$ and $L^F$ workers (consumers). For simplicity, assume that consumers have identical preferences across the two countries and that there is no labor mobility. As in the closed-economy setting, the demand for good $i$ in country $l$ ($l = \{H, F\}$) is given by

$$q^l_i = L^l q^c_i = \frac{\alpha L^l}{\eta N^l + \gamma} - \frac{L^l}{\gamma} p^l_i + L^l z_i + \frac{\eta N^l L^l}{\eta N^l + \gamma} \bar{p}^l - \frac{1}{2} \frac{\eta N^l L^l}{\eta N^l + \gamma} \bar{z}^l$$

where $p^l_i$ and $q^l_i$ is the price of good $i$ and quantity demanded in country $l$, respectively. Average price and quality in country $l$ are given by $\bar{p}^l$ and $\bar{z}^l$. There are $N^l$ firms selling in country $l$. These are both domestic firms and foreign exporters.

\subsection{Firms:}

As in the closed economy setting, a firm chooses whether or not to produce, and the level of quality for each market. The firm now has the option to export. There is a positive cost to exporting, so not all firms choose to export. A firm that exports sets different levels of quality and different prices for the domestic and the foreign markets.\textsuperscript{19} The delivery cost of a unit with cost $c$ to country $l$ is $\tau^l c$.\textsuperscript{20} Let $p^l(c)$ and $q^l(c)$

\textsuperscript{18}For example, in Arkolakis et. al. (2010, 2012a, 2012b) the authors show that in a very broad class of trade models, welfare gains from trade are limited. However, in all the models considered, competition does not affect the scope for quality differentiation. Failing to account for this channel and for the fact that competition can raise average quality, may offer an explanation to the surprising results in their work.

\textsuperscript{19}For empirical evidence on firms setting different levels of quality across export destinations, see Manova and Zhang (2012).

\textsuperscript{20}An alternative specification is to consider a per-unit export cost, where the cost of delivering a unit with cost $c$ to country $l$ is $\tau^l + c$. While we do not report this specification here, the solution is available by the author upon request.
be the domestic level of the profit maximizing price and quantity, respectively. The operating profits from
domestic and foreign sales are given by

\[ \pi_D^I(c, z) = [p_D^I(c, z) - c] q_D^I(c, z) \]  
(18a)

\[ \pi_X^I(c, z) = [p_X^I(c, z) - \tau^h c] q_X^I(c, z) \]  
(18b)

The profit maximizing prices and quantities must satisfy

\[ q_D^I(c, z) = \frac{L^I}{\gamma} [p_D^I(c, z) - c] \]  
(19a)

\[ q_X^I(c, z) = \frac{L^h}{\gamma} [p_X^I(c, z) - \tau^h c] \]  
(19b)

The production cutoffs are defined as

\[ c_D^I = \sup \{ \pi_D^I(c) > 0 \} = p_D^I \]  
(20a)

\[ c_X^I = \sup \{ \pi_X^I(c) > 0 \} = \frac{p_X^I}{\tau^h} \]  
(20b)

Combining the cutoffs conditions for the two countries, it is easy to show that \( c_X^I = c_D^I/\tau^I \). The optimal
price and quantity for the domestic and the foreign market can now be expressed as functions of the cutoff
cost thresholds.

\[ p_D^I(c, z) = \frac{1}{2} (c_D^I + c) + \frac{\gamma}{2} z_D^I \]  
(21a)

\[ p_X^I(c, z) = \frac{\tau^h}{2} (c_X^I + c) + \frac{\gamma}{2} z_X^I \]  
(21b)

\[ q_D^I(c, z) = \frac{L^I}{2\gamma} (c_D^I - c) + \frac{L^I}{2} z^I_D \]  
(21c)

\[ q_X^I(c, z) = \frac{L^h}{2\gamma} \tau^h (c_X^I - c) + \frac{L^h}{2} z^I_X \]  
(21d)
Given prices and quantities, operating profits for the firm in each market are given by

\[ \pi^l_D(c, z) = \frac{L^l}{4\gamma} (c^l_D - c)^2 + \frac{L^l}{2} (z^l_D)(c^l_D - c) + \frac{\gamma L^l}{4} (z^l_D)^2 - \theta(z^l_D)^2 \]  

(22a)

\[ \pi^l_X(c, z) = \frac{L^h}{4\gamma} (\tau^h)^2(c^l_X - c)^2 + \frac{L^h}{2} (z^l_X)(c^l_X - c) + \frac{\gamma L^h}{4} (z^l_X)^2 - \theta(z^l_X)^2 \]  

(22b)

The optimal levels of product quality \( z^* \) for the domestic and the foreign market are given by

\[ z^*_D(c) = \lambda_D^l(c^l_D - c) \quad \text{(23)} \]

\[ z^*_X(c) = \lambda_X^l(\tau^h)(c^l_X - c) \quad \text{(24)} \]

where \( \lambda_D^l = L^l/(4\theta^l - L^l\gamma) \), and \( \lambda_X^l = L^h/(4\theta^l - L^h\gamma) \) are the scopes for quality differentiation in the domestic and in the foreign markets, from the point of view of a firm operating in country \( l \), which faces a cost of innovation \( \theta^l \). To ensure that all qualities are positive, we assume that all \( \lambda \)'s are positive.

We complete the model by substituting the optimal value of \( z^* \) into (21a) and (22a).
3.3 Free-Entry Condition

At equilibrium, the expected profit of a firm is 0. Therefore

\[ f_E = \int_{0}^{c_l} \pi_D(c) dG(c) + \int_{0}^{c_l} \pi_X(c) dG(c) \]  

(31)

We assume that cost draws come from the same Pareto parameterization as in the closed economy model. Given the parameterization, the expressions for the profits in the domestic and foreign markets, and the fact that \( c_X^h = c_D^l / \tau^l \), we can now re-write the free-entry condition as

\[ L^l (1 + \gamma \lambda_D^l) (c_D^l)^{k+2} + L^h (1 + \gamma \lambda_X^h) (c_D^l)^{k+2} \rho^h = \gamma \varphi \]  

(32)

where \( \phi = 2e^k (k+1)(k+2)f_E \), \( \rho^h = (\tau^h)^{-k} \). We can write a similar condition for the foreign country. This yields a system of two equations and two unknowns which we solve for \( c_D^l \) and \( c_D^h \).

\[ c_D^l = \left\{ \frac{\gamma \varphi}{L^l} \left[ \frac{(1 + \gamma \lambda_D^l) - \rho^h (1 + \gamma \lambda_X^h)}{(1 + \gamma \lambda_D^l) + \rho \rho^h (1 + \gamma \lambda_X^h)} \right] \right\}^{1/(k+2)} \]  

(33)

Country characteristics of both trading partners, along with the trade barriers, determine the cost threshold.

We can now express the aggregate measures in the economy as functions of the cost threshold \( c_D^l \). As in the case of the closed economy, these measures will also be functions of the scope for quality differentiation in each market, from the perspective of the domestic and the foreign firms facing \( \theta^l \) and \( \theta^h \) costs of innovation, respectively.

We proceed in two steps. First, we compute the average price, quality, and markups for the domestically produced and for the foreign produced bundles of goods consumed at home. Then, we aggregate across the two bundles to obtain the average price, quality, and markups for all goods, domestic and foreign, consumed at home.

Let \( N_D^l \) and \( N_X^h \) represent the domestic and foreign firms selling in the domestic country, respectively, and \( N^l = N_D^l + N_X^h \) the total number of firms servicing the domestic market. The average prices of domestic, of foreign, and of all goods consumed in the domestic market are given by:
where $\bar{D} = (N^l_D\lambda^l_D + N^h_X\lambda^h_X)/N^l$ is the (weighted) average scope for quality differentiation of all $N^l$ firms servicing the domestic market. Equations (35) and (36) show that that the price distribution in country $l$ of domestic firms and exporters producing in $h$ and servicing the domestic market will be the same if and only if $\theta^l = \theta^h$.\footnote{\text{21} Since $\lambda^l_D = L^l/(4\theta^l - L^l)$ and $\lambda^h_X = L^h/(4\theta^h - L^l)$, the two measures will be the same if and only if $\theta^l = \theta^h$.} But in general, the two price distributions will differ. More specifically, the basket of imported goods from $h$ will cost more if the ability to innovate in $h$ is higher than in $l$, i.e. if $\theta^l > \theta^h$. It will also be of higher quality as we show next. The average price $\bar{p}_D$ of the consumption basket in the domestic market will now be a function of the (weighted) average scope for quality differentiation between the $N^l_D$ domestic and $N^h_X$ foreign firms servicing the domestic market. In a sense, $\bar{\lambda}_D$ represents the effective scope for quality differentiation in the domestic market. What is nice here, is that we can now express all aggregate measures as functions of the effective scope, and then consider how trade openness affects the economy through its impact on $\bar{\lambda}_D$. We elaborate more on this next, but first we complete the presentation of all aggregate measures.

Average quality and markups are given by:

\begin{align*}
\bar{\pi}^l_D &= \frac{2k + 1 + \gamma \lambda^l_D}{2k + 2} c_D \\
\bar{\pi}^h_X &= \frac{k + 1 + \gamma \lambda^h_X}{2k + 2} c_D \\
\bar{\pi}_D &= \frac{2k + 1 + \gamma \lambda^l_D}{2k + 2} c_D
\end{align*}

(34) (35) (36)

and
\[ p^*_D = \frac{1}{2} \left[ (1 + \gamma \lambda_D^l) \left( \frac{1}{k+1} \right) c_D^l \right] \] (40)

\[ p^*_X = \frac{1}{2} \left[ (1 + \gamma \lambda_X^l) \left( \frac{1}{k+1} \right) c_D^l \right] \] (41)

\[ p^*_D = \frac{1}{2} \left[ (1 + \gamma \lambda_D) \left( \frac{1}{k+1} \right) c_D^l \right] \] (42)

The total number of firms, both domestic and foreign, servicing the domestic market is

\[ N^l = \frac{2(k + 1)\gamma (\alpha - c_D^l)}{n} \] (43)

and total welfare is given by

\[ U^l = 1 + \frac{1}{2\eta} (\alpha - c_D^l) \left[ \alpha - \frac{k + 1}{k + 2} c_D^l \right] + \gamma \lambda_D (\alpha - c_D^l) \left[ \frac{12\alpha - 2(c_D^l - 3\alpha)k - (2 + 3\gamma \lambda_D) c_D^l}{2(k + 2)\eta} \right] \] (44)

With the open economy model fully specified, we proceed to discuss some of the key implications of the theory. Since the impact of trade in a setting with heterogeneous firms and endogenous quality choice has been thoroughly analyzed in the past (see Melitz and Ottaviano 2008), we highlight the implications that are unique in the context of this theory.

The transition from autarky to trade is equivalent to an increase in market size and operates through two channels: it induces more competition and it affects the scope for quality differentiation.\(^{22}\) Hence, the intuition from our earlier discussion on the impact of market toughness on aggregate measures in a closed economy setting also applies in the discussion of trade openness.

Trade-induced competition suppresses quality, price, and markups, while trade-induced innovation raises quality, price, and markups. How these aggregate measures respond to trade openness now depends on how similar or dissimilar the trading partners are with respect to their innovation capacities. As seen from equations (37), (40), and (43), trading with a developed country raises average quality, price, and markups because it raises the effective scope for quality differentiation of all firms servicing the domestic market. Trading with a developing country lowers the effective scope and lowers these measures. For trading partners that are similar in their innovation capacity, which is proxy by the inverse of \(\theta^l\) and \(\theta^h\), the changes

\(^{22}\) It becomes identical to an increase in market size if no trade barriers are present.
in these measures will not be substantial since $\overline{\lambda}_D$ will not differ much from $\lambda'_D$. However, for countries that are farther apart in their innovation capacities, the change in the effective scope will be substantial, and so will the change in the aggregate measures.

Another related, yet important implication of the model is that imported goods from developed countries will be of higher quality and will cost more than imported goods from developing countries. To see this, consider what equation (36) implies for the average price of a bundle of imported goods originating from country $h$ relative to the price of a bundle of imported goods from country $i$. Foreign firms in both countries will be subject to the same domestic cost threshold $c^l_D$ and the same market size $L_l$. Therefore, what will drive differences in the price of the two bundles of imported goods is only differences in the innovation capacities across the two exporters, namely $\theta^h$ and $\theta^i$. The country with the highest innovation capacity will export a more expensive bundle of goods. A similar argument based on equation (39) shows that the export bundle of the country with higher innovation capacity will also be of higher quality. And if we were to generalize this to $N$ countries exporting to $l$, then we would see that differences in the innovation capacity across countries are sufficient to explain differences in the average quality and price of each bundle of exported goods. The fact that innovation capacity of the exporter is more important than its size when explaining patterns of trade is emphasized by Schott (2004) who documented that China’s exports to the US are cheap and of low quality.

4 Conclusion

We presented a simple and tractable trade model of heterogeneous firms, endogenous quality choice, and endogenous markups. A key feature of the model is that all firm performance measures and all economic aggregates can be written as functions of the cost threshold and the scope for quality differentiation. By considering how changes in economic conditions affect both the threshold and the scope we come up with a very rich and empirically relevant theory.

Indeed, we provide a supply-side explanation for a very rich set of facts related to (i) firm heterogeneity, (ii) $^23$Although in this case $c^l_D$ will be different from the solution in (34) since there are more than one countries exporting to $l$, all exporters will still face the same domestic cost threshold.
quality heterogeneity, (iii) markups heterogeneity, (iv) heterogeneity in the response of firms to competition, and (v) heterogeneity in the sign and magnitude of the correlations between various firm performance measures. We also make some novel theoretical predictions about aggregate economic measures.

Besides matching firm behavior and patterns of trade, there are several other areas where we think such a model is relevant. When making inferences about international price differences, Feenstra and Romalis (2012) and Feenstra and Weinstein (2012) emphasize the need to account for variations in quality and in markups, respectively, across sectors and countries. However, the former study assumes that markups are constant, while the latter study does not consider quality-adjusted prices. Such studies can benefit from a framework where both quality and markups vary.

Another area where we see the model being relevant is on the impact of trade openness and trade liberalization on the relative demand for high- versus low-skilled workers and on the dynamics of the wage gap. Abstract for a minute from the assumption of homogeneous workers, and consider that quantity (the first term of the cost function) is produced from low-skilled workers, while quality (the second term) is produced from high-skilled. As economic conditions change and quality ladders pivot, the most productive firms raise quality and price, while the least productive lower quality and prices. Consequently, the relative demand for high- versus low-skilled workers will vary across firms within sectors, across sectors within a country, and across countries as the scopes for quality differentiation move. Hence, expanding our model to account for labor heterogeneity is a very promising direction, and one that we are currently in pursuit.
References


[38] Hallak, Juan-Carlos, and Jagadeesh Sivadasan, “Firms’ Exporting Behavior under Quality Constraints," Research Seminar in International Economics No. 628 (2011)


A Variance of firm performance measures

Let \( \sigma_y^2 = \frac{1}{G(c_D)} \int_0^{c_D} [y(c) - \bar{y}]^2 dG(c) \) denote the variance of a firm performance measure \( y(c) \). Then,

\[
\sigma_y^2 = \frac{1}{4}(-1 + \gamma \lambda)\sigma_c^2 \tag{A.1}
\]
\[
\sigma_\mu^2 = \frac{1}{4}(1 + \gamma \lambda)\sigma_c^2 \tag{A.2}
\]
\[
\sigma_q^2 = \frac{L^2}{4\gamma^2}(1 + \gamma \lambda)^2\sigma_c^2 \tag{A.3}
\]
\[
\sigma_r^2 = \frac{L^2}{16}(1 + \gamma \lambda)\sigma_c^2\psi \tag{A.4}
\]

where \( \sigma_c^2 = \left[k/((k + 1)^2(k + 2))\right] c_D \) and \( \psi = (3 + 5k^2 + 3 + 5k^2 + 7k + 4k^2\gamma \lambda + 5k\gamma^2 \lambda^2 + k^3 + 14k\gamma \lambda + 11\gamma^2 \lambda^2 + 10\gamma^2 \lambda^2)/(\gamma^2(1 + k)^2(k + 3)) > 0. \)
Figure 1 – Quality Ladders

i. Homogeneous Sector

ii. Heterogeneous Sector
Figure 2 – Quality Ladders and Market Toughness
Figure 3 – Aggregate Variables and Market Competition

— Melitz-Ottaviano
--- Developing
— Developed